# Chapter 7

# Baroclinic instability and midlatitude storms

## 7.1 Three-dimensional geostrophic flow

### 7.1.1 In geometric coordinates $(x, y, z)$

In Cartesian, geometric coordinates, the equations of motion and of hydrostatic balance are

$$\begin{aligned} \frac{du}{dt} - fv &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + \mathcal{F}_x , \\ \frac{dv}{dt} + fu &= -\frac{1}{\rho}\frac{\partial p}{\partial y} + \mathcal{F}_y , \\ \frac{\partial p}{\partial z} &= -g\rho , \end{aligned} \tag{7.1}$$

where $(\mathcal{F}_x, \mathcal{F}_y)$ are the $(x, y)$ components of friction. The continuity equation is

$$\frac{d\rho}{dt} + \rho\nabla.\mathbf{u} = 0 . \tag{7.2}$$

For small Rossby number ($U/fL \ll 1$, where $U$ and $L$ are magnitudes for the flow and for spatial scales), the wind can be determined from *geostrophic balance*:

$$\mathbf{u} = \frac{1}{f\rho}\widehat{\mathbf{z}} \times \nabla p , \tag{7.3}$$

or, in its components,

$$\begin{aligned} u &= -\frac{1}{f\rho}\frac{\partial p}{\partial y} ; \\ v &= \frac{1}{f\rho}\frac{\partial p}{\partial x} ; \\ w &= 0 . \end{aligned} \tag{7.4}$$

Note that

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 :$$

the geostrophic wind is nondivergent[1], which means the geostrophic flow is quasi-2D, and has some parallels with barotropic (precisely 2D) flow.

Since $\mathbf{u} \cdot \nabla p = 0$, the flow is *normal to the pressure gradient*, along the isobars. Thus, the isobars are *streamlines of the geostrophic flow.* In fact, from (7.4) we can define a *geostrophic streamfunction*, $\psi = p/(f\rho)$, which has the same properties as barotropic streamfunction.

### 7.1.2 In pressure coordinates (x,y,p)

In pressure coordinates (which are more useful for compressible atmospheres than height coordinates) the equations become

$$\begin{aligned} \frac{du}{dt} - fv &= -g\frac{\partial z}{\partial x} + \mathcal{F}_x , \\ \frac{dv}{dt} + fu &= -g\frac{\partial z}{\partial y} + \mathcal{F}_y , \\ \frac{\partial z}{\partial p} &= -\frac{1}{g\rho} , \end{aligned} \tag{7.5}$$

where the $x-$ and $y-$ derivatives should be understood as applying at constant pressure. One of the great simplifications of pressure coordinates is that the continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = \nabla_p \cdot \mathbf{u} = 0 , \tag{7.6}$$

where $\omega = dp/dt$ is the pressure coordinate equivalent of vertical velocity.

[1] This is strictly true only if variations in $f$ are negligible, which means that the length scale of the motions must be much less than the Earth's radius.

Now geostrophic balance (7.3) becomes, in pressure coordinates,

$$\mathbf{u} = \frac{g}{f}\widehat{\mathbf{z}_p} \times \nabla_p z \, , \tag{7.7}$$

where $\widehat{\mathbf{z}_p}$ is the upward unit vector in pressure coordinates and $\nabla_p$ denotes the gradient operator in pressure coordinates. In component form,

$$(u, v) = \left( -\frac{g}{f}\frac{\partial z}{\partial y}, \frac{g}{f}\frac{\partial z}{\partial x} \right) \, .$$

Note that, like $p$ contours on surfaces of constant $z$, $z$ contours on constant $p$ are streamlines of the geostrophic flow.

### 7.1.3 Thermal wind balance

. Taking the $p$-derivative of the $x$-component of (7.7) gives

$$\frac{\partial u}{\partial p} = -\frac{g}{f}\frac{\partial^2 z}{\partial p \partial y} = -\frac{g}{f}\left(\frac{\partial}{\partial y}\left[\frac{\partial z}{\partial p}\right]\right)_p = \frac{1}{f}\frac{\partial}{\partial y}\left(\frac{1}{\rho}\right)_p \, .$$

Since $1/\rho = RT/p$, its derivative *at constant pressure* is

$$\frac{\partial}{\partial y}\left(\frac{1}{\rho}\right)_p = \frac{R}{p}\left(\frac{\partial T}{\partial y}\right)_p \, ,$$

whence

$$\frac{\partial u}{\partial p} = \frac{R}{fp}\left(\frac{\partial T}{\partial y}\right)_p \, . \tag{7.8}$$

Similarly, for $v$ we find

$$\frac{\partial v}{\partial p} = -\frac{R}{fp}\left(\frac{\partial T}{\partial x}\right)_p \, . \tag{7.9}$$

Thus, horizontal gradients of temperature must be accompanied by vertical gradients of wind.

### 7.1.4 Thermodynamic equation

In eq. (3.14), we had

$$\frac{dT}{dt} - \frac{1}{\rho c_p}\frac{dp}{dt} = \frac{J}{\rho c_p} \, ,$$

where $J$ is the **diabatic heating rate** per unit volume. Now, just as in geometric coordinates where the natural definition of vertical velocity is $w = dz/dt$, in pressure coordinates "vertical velocity" becomes[2] $\omega = dp/dt$. Then (3.14) can be written

$$\frac{dT}{dt} - \frac{1}{\rho c_p}\omega = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} - \omega S = \frac{J}{\rho c_p}, \tag{7.10}$$

where $S = \partial T/\partial p - 1/\rho c_p < 0$ for a stable atmosphere, and $J$ is the diabatic heating rate per unit volume. Note that $J = 0$ for adiabatic motions.

Equivalently, defining potential temperature $\theta = T\,(p_0/p)^\kappa$, as in (3.15), (7.10) can be written

$$\frac{d\theta}{dt} = \left(\frac{p_0}{p}\right)^\kappa \frac{J}{\rho c_p}. \tag{7.11}$$

For adiabatic motions ($J = 0$), $\theta$ is conserved following the flow.

## 7.2 Structure of synoptic storm systems

The typical midlatitude synoptic storm system, such as those seen in Fig. 6.1, are mobile systems of both low and high pressure (though only the low pressure systems are usually associated with storms) that dominate the meteorology of the lower atmosphere, especially in winter. A typical northern hemisphere pattern may look like that shown in Fig. 7.1. Typical length

[2] Since $p$ varies most strongly in the vertical,

$$\begin{aligned}\frac{dp}{dt} &\equiv \frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z} \\ &\simeq w\frac{\partial p}{\partial z} = -wg\rho\,,\end{aligned}$$

assuming hydrostatic balance. So $\omega$ and $w$ are opposite in sign—*e.g.*, $\omega < 0$ is upward motion (toward lower pressure). Note also that

$$\begin{aligned}S &= \frac{\partial T}{\partial p} - \frac{1}{\rho c_p} \\ &= -g\rho\left(\frac{\partial T}{\partial z} + \frac{g}{c_p}\right).\end{aligned}$$

scales are a few hundred km (with high pressure systems being typically larger than low pressure systems). From (7.4), it follows that the geostrophic flow, along the pressure contours, is *cyclonic* (in the same sense as the Earth's rotation) around the low and *anticyclonic* around the high pressure center.

Surface pressure is always within 10% of 1000hPa; in middle latitudes, typical storms may have pressure anomalies of 20hPa (cyclones) or 10hPa (anticyclones). If we regard each eddy as circular, then we may represent, crudely, the pressure structure (departure from 1000hPa) of a cyclonic eddy as $p' \simeq P_0 \exp(-r^2/2L^2)$, where $r$ is the distance from the center, and $L$ the radius at which $p'$ falls off by $1/\sqrt{e}$ from its central value. The azimuthal component of wind is, from (7.4),

$$\begin{aligned} u &= \frac{1}{f\rho}\frac{\partial p'}{\partial r} \\ &= -\frac{P_0}{f\rho}\frac{r}{L^2}\exp(-r^2/2L^2) . \end{aligned}$$

The maximum wind is at $r = L$, where

$$|u|_{\max} = \frac{P_0}{f\rho L}e^{-\frac{1}{2}} .$$

Air has STP has density 1.293 kg $m^{-3}$; at 45° latitude, $f \simeq 1.0 \times 10^{-4} s^{-1}$. Hence, using $P_0 = 2000$Pa, and a size $L = 500$km, we find

$$|u|_{\max} \simeq 20 \text{ms}^{-1} .$$

This is a typical maximum wind in the lower *free* troposphere. Within the frictional boundary layer (where, of course, we live) surface friction slows the flow, and makes it *sub-geostrophic*, spiraling into low pressure cyclones and out of high pressure anticyclones, as in Fig. 7.2. The low-level inflow into cyclones produces, through "Ekman pumping" in the Ekman boundary layer, upwelling within the cyclones, which are therefore associated with clouds and rain; in anticyclones, descending air makes for clear skies. But note that most of the "weather" associated with cyclones is associated with thermal fronts embedded within the cyclone; we shall discuss these later.

## 7.3 Cyclogenesis and energetics

Where do these synoptic systems come from? They are mobile, and thus not attached to any surface features, so it seems unlikely that they are pro-

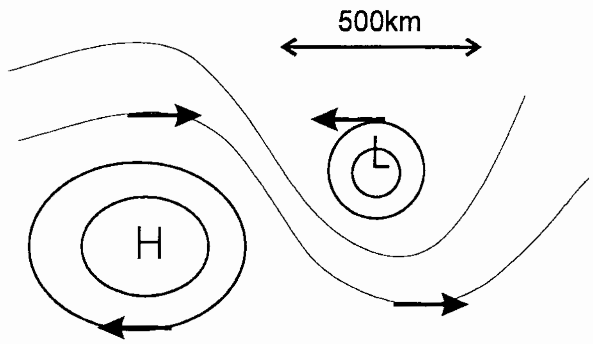


Figure 7.1: Schematic of the surface isobars around synoptic systems.

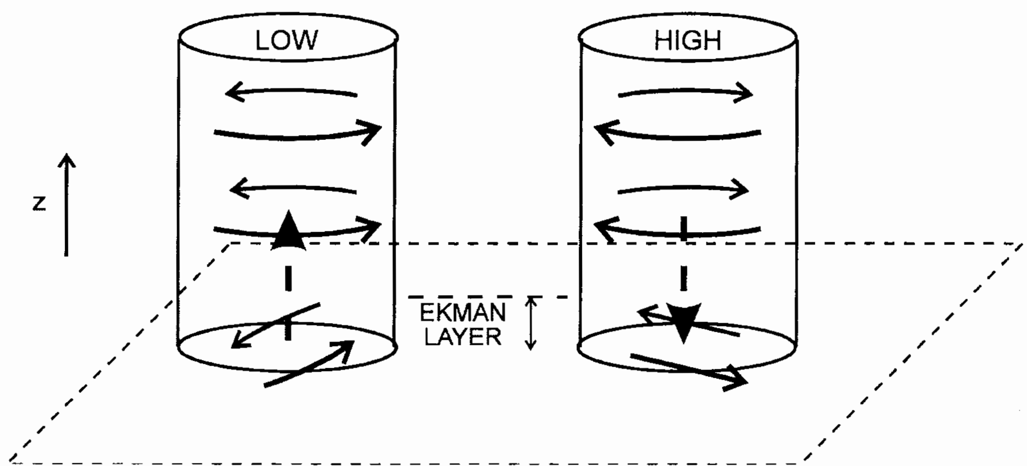


Figure 7.2: Schematic of Ekman inflow (in low pressure systems) and outflow (in high pressure systems).

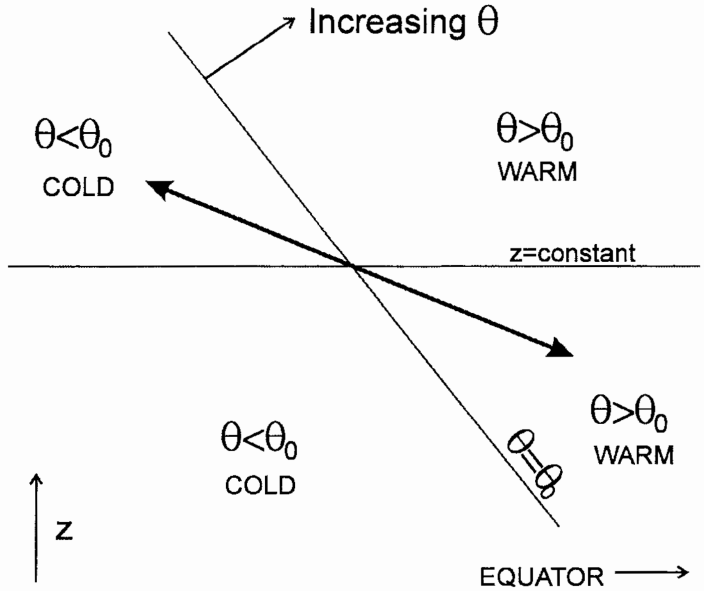


Figure 7.3: The "wedge of instability."

duced by external forcing. Rather, they are produced by a process known as *baroclinic instability.* The presence of horizontal temperature gradients in the atmosphere implies the existence of *available potential energy*, since the isentropes (surface of constant $\theta$) can be re-arranged to reduce the potential energy, as shown in Fig. 7.3. Exchanging air in the direction of the arrow will reduce the potential energy by moving warm (light) air upward and cold (heavy) air downward. The potential energy lost must appear as kinetic energy of the motion.

We can formalize this by considering the development of eddies on a basic state that is initially independent of $x$; for simplicity (in fact, to avoid some unnecessary complications), we assume that the basic flow is in fact exactly zonal, and that the flow is a function of $p$ only. Note, of course, from thermal wind balance (7.8), that the basic state temperature must then be a function of $y$ as well as $p$:

$$\begin{aligned} (u, v, w) &= (U_0(p), 0, 0) \; ; \\ T &= T_0(y, p) \; ; \\ \theta &= \theta_0(y, p) \; . \end{aligned} \tag{7.12}$$

We now consider perturbations to this state. To keep things simple, we will assume the perturbations to be small so that we can linearize in the usual way. Of course, the real, fully developed weather systems we are interested in are not of small amplitude. If they grow through a linear instability, they grow from infinitesimal amplitude (when our linear assumptions are justifiable), reaching finite amplitude only as they mature. Detailed calculations (which are beyond the scope of what we are trying to do here) show that, for a typical midlatitude atmospheric state, waves with wavelengths of around 1000km will grow the fastest, with growth times ($e$-folding times) of typically 2-3 days.

The more limited question we are going to ask is where these systems get their energy from in the first place, and what characteristics they must have to allow them to extract energy from the basic state. Perturbing the state (7.12), therefore, and taking the inviscid eqns. of motion (7.5),

$$\begin{aligned} \frac{\partial u'}{\partial t} + U_0 \frac{\partial u'}{\partial x} + - f v' &= -g \frac{\partial z'}{\partial x} , \\ \frac{\partial v'}{\partial t} + U_0 \frac{\partial v'}{\partial x} + f u' &= -g \frac{\partial z'}{\partial y} , \\ \frac{\partial z'}{\partial p} &= -\frac{RT'}{gp} . \end{aligned} \tag{7.13}$$

Now, the kinetic energy of the perturbation motions per unit volume is $\rho \left( u'^2 + v'^2 \right)$; therefore their globally integrated K.E. is

$$\begin{aligned} K &= \int \int \int_0^\infty \rho \left( u'^2 + v'^2 \right) \, dx \, dy \, dz \\ &= \frac{1}{g} \int \int \int_0^{p_0} \left( u'^2 + v'^2 \right) \, dx \, dy \, dp . \end{aligned} \tag{7.14}$$

Now, we can form an equation for $dK/dt$ by taking $u' \times$the first of (7.13)+$v' \times$ the second:

$$\left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \frac{1}{2} \left( u'^2 + v'^2 \right) = -g \left( u' \frac{\partial z'}{\partial x} + v' \frac{\partial z'}{\partial y} \right) .$$

Using the continuity equation, we can write

$$\begin{aligned} \left( u' \frac{\partial z'}{\partial x} + v' \frac{\partial z'}{\partial y} \right) &= \mathbf{u}' \cdot \nabla_p z' - \omega' \frac{\partial z'}{\partial p} \\ &= \nabla_p \cdot \left( \mathbf{u}' z' \right) - \omega' \frac{\partial z'}{\partial p} . \end{aligned}$$

Using hydrostatic balance, $\partial z'/\partial p = RT'/gp$, so

$$\frac{1}{g}\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\frac{1}{2}\left(u'^2+v'^2\right)=\nabla_p\cdot(\mathbf{u}'z')-\frac{R}{gp}\omega'T' .$$

Integrating over the whole system,

$$\frac{1}{g}\iiint\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\frac{1}{2}\left(u'^2+v'^2\right)dx\,dy\,dp = \iiint \nabla_p\cdot(\mathbf{u}'z')\;\;dx\,dy\,dp - \iiint\frac{R}{gp}\omega'T'\;dx\,dy\,dp .$$

But

1. $\iiint \frac{\partial}{\partial t}\left[\frac{1}{2}\left(u'^2+v'^2\right)\right]dx\,dy\,dp = \frac{dK}{dt}$ ;

2. $\iiint \frac{\partial}{\partial x}\left[\frac{1}{2}\left(u'^2+v'^2\right)\right]dx\;dy\;dp = \iint\left[\frac{1}{2}\left(u'^2+v'^2\right)\right]_{x_1}^{x_2}\;dy\;dp$, where $x_1$, $x_2$, are the limits of integration. But, since the system is periodic (360° is the same as 0°), $[\_]_{x_1}^{x_2}=0$. Hence $\iiint \frac{\partial}{\partial x}\left[\frac{1}{2}\left(u'^2+v'^2\right)\right]dx\;dy\;dp=0$.

3. $\iiint \nabla_p\cdot(\mathbf{u}'z')\;\;dx\;dy\;dp=\iint \mathbf{u}'z'\cdot\mathbf{n}\;dA$, where the integral is over the area bounding the system (the entire atmosphere) and $\mathbf{n}$ is the unit normal to the boundary. Since the only boundaries are the Earth's surface $p=p_s$ and the top of the atmosphere $p=0$,

$$\iiint\nabla_p\cdot(\mathbf{u}'z')\;\;dx\;dy\;dp=\iint\left[\omega'z'\right]_{p=p_s}\;dx\;dy-\iint\left[\omega'z'\right]_{p=0}\;dx\;dy .$$

But, at the top of the atmosphere, $p=0$ and $\omega'=0$; at the surface, which is geometrically fixed $z'=0$. So $\iiint\nabla_p\cdot(\mathbf{u}'z')\;\;dx\;dy\;dp=0$.

Therefore

$$\frac{dK}{dt}=-\iiint\frac{R}{gp}\omega'T'\;dx\;dy\;dp . \tag{7.15}$$

Hence, the disturbance kinetic energy can grow only if, on average $\omega'T'<0$ (*upward heat flux*). Since upward motion (toward lower pressure) means $\omega<0$, this means that warm (light) air must rise and cold (dense) air sink in a developing storm.

That much may seem obvious, but there is more. Consider now the thermodynamic equation; assuming adiabatic motion[3], $d\theta/dt = 0$. The perturbed equation is therefore

$$\frac{\partial \theta'}{\partial t} + U_0 \frac{\partial \theta'}{\partial x} + v' \frac{\partial \theta_0}{\partial y} + \omega' \frac{\partial \theta_0}{\partial p} = 0 \; .$$

Multiplying by $\theta'$ and integrating over the atmosphere:

$$\iiint \left( \frac{\partial}{\partial t} + U_0 \frac{\partial}{\partial x} \right) \frac{1}{2} \theta'^2 \; dx \; dy \; dp = - \iiint \left[ v'\theta' \frac{\partial \theta_0}{\partial y} + \omega'\theta' \frac{\partial \theta_0}{\partial p} \right] \; dx \; dy \; dp \; .$$

Now, as above, it follows that

$$\begin{aligned} \iiint U_0(p) \frac{\partial}{\partial x} \left[ \frac{1}{2} \theta'^2 \right] \; dx \; dy \; dp &= \int \int \left\{ \int \frac{\partial}{\partial x} \left[ \frac{1}{2} \theta'^2 \right] \; dx \right\} U_0(p) \; dy \; dp \\ &= \int \int \left[ \frac{1}{2} \theta'^2 \right]_{x_1}^{x_2} U_0(p) \; dy \; dp \\ &= 0 \; , \end{aligned}$$

so

$$\frac{d}{dt} \iiint \frac{1}{2} \theta'^2 \; dx \; dy \; dp = - \iiint \left[ v'\theta' \frac{\partial \theta_0}{\partial y} + \omega'\theta' \frac{\partial \theta_0}{\partial p} \right] \; dx \; dy \; dp \qquad (7.16)$$

Note that, if the disturbance is to grow from infinitesimal amplitude, the l.h.s must be positive (since $\theta'^2$ is positive definite).

Thus, from the two constraints (7.15) and (7.16), we have that, on average, $\omega'\theta' < 0$ and $\mathbf{u}'\theta' \cdot \nabla_p \theta_0 < 0$: so the vertical component must be upward BUT the vector component must be *downgradient*, toward lower basic state $\theta_0$. In order to achieve this—see Fig. 7.4)—we need the vector $(v'\theta', \omega'\theta')$ to lie within the "wedge of instability" between the horizontal and mean isentropic surfaces, as previously depicted in Fig. 7.3. Fig. 7.4 has been drawn on the easily defensible assumptions that the atmosphere is statically stable against convection—so that $\theta_0$ increases upward—and that the basic state

[3] In fact, of course, rain—consequent on condensation and on release of latent heat—is a feature of intense storms, and thus the motions will not be adiabatic. However, this effect is not essential to the mechanism of storm development.

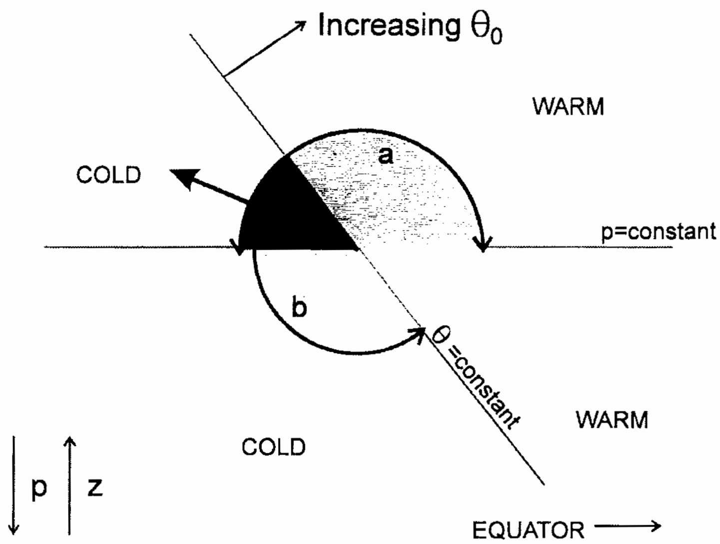


Figure 7.4: The "wedge of instability." The heat flux ($v'\theta'$, $\omega'\theta'$) must be both upward (region $a$) and downgradient (region $b$) and thus must lie within the overlap of the two regions, as shown by the arrow.

temperature decreases toward the pole—so that $\theta_0$ does the same. Hence the gradient vector $\nabla\theta_0$ is as drawn.

As depicted in Fig. 7.4, the average of the *poleward heat flux*, $v'\theta'$, must be poleward. This follows from eqs. (7.15) and (7.16):

1. $\partial\theta_0/\partial p < 0$ ($\theta$ increases upward) in a stable atmosphere; and

2. $\theta' = T'(p_0/p)^\kappa$, whence $\omega'\theta' = \omega'T'(p_0/p)^\kappa$, and we saw above that the integral of the latter must be negative; so

3. the second term on the r.h.s. of (7.16) must be negative (allowing for the minus sign).

4. So the wave can grow only if the first term on the r.h.s. is sufficiently positive.

5. Since $\partial\theta_0/\partial y < 0$ (temperature decreases poleward), growing disturbances must have $v'\theta' > 0$.

Therefore, growing disturbances must *transport heat poleward* in order to extract energy from the basic state. Since the reason the basic state has available potential energy (APE) for the disturbance to extract is the presence of a horizontal temperature gradient, it is not surprising that the disturbances must transport heat down this gradient. By transporting heat poleward, the disturbances tend to warm the higher latitudes, thus reducing the APE. The lost APE appears, of course, as KE of the disturbances.

## 7.4 Vertical structure of growing disturbances

The simplest growing disturbances can be represented as being wavelike in the longitudinal direction, with height perturbation something like

$$z'(x,p) = \mathcal{R}e\ F(p)\ e^{i(kx-\omega t)}$$

(remember $\omega$ is now complex when the wave is growing). We have neglected any $y$-variation here; this is not particularly realistic, but it suffices to illustrate the point.

Suppose first that there is *no vertical phase tilt* of the disturbance, as depicted in Fig. 7.5. The 'L' and 'H' denote the locations of low and high

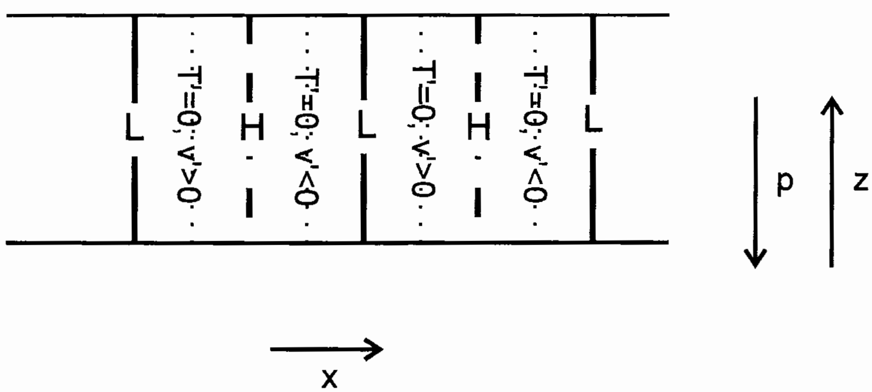


Figure 7.5:

height ($z'$) perturbations, respectively. The dotted lines show where $z' = 0$ at all heights—since we have specified no phase tilt. Then from the hydrostatic eq., (7.5), it follows that $T' = 0$ there (where $z' = 0$) also. But, geostrophic balance, eq. (7.7), tells us that $v' = (g/f)\partial z'/\partial x$ is an *extremum* at this location. Therefore $v'$ and $T'$ are out of phase (in quadrature, in fact) so that, on average, $v'T' = 0$. This clearly cannot be the structure of a growing disturbance.

Consider now a disturbance whose phase slopes westward with height; see Fig. 7.6. The maximum northward (/southward) flow is still to the

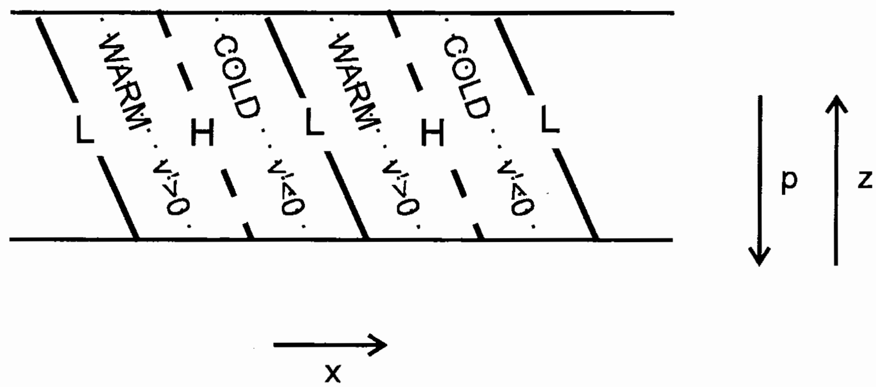


Figure 7.6:

east of the low (/high) height perturbation (northern hemisphere), but now there is a temperature anomaly there. Where $v' > 0$, $z'$ increases with $z$ (decreases with $p$), so the temperature perturbation is *warm* there; so warm air is moving north. Similarly, 180° further east, cold air is moving south. Hence $v'T' > 0$ for this configuration, showing that this wave structure will lead to growth.

If the disturbance is tilted eastward with height, one finds $v'T' < 0$, so there is no growth in this case.

An example of this is shown in Fig. 7.7. On 19 Dec 1964, a weak surface low lies to the east of an upper level (500hPa) trough (marked 'A' on the figure)—so the low tilts westward with height. This is favorable for growth, as illustrated by the explosive development that followed over the next 12

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hrs.

## 7.5 Fronts

Even though baroclinic systems get their energy by reducing the overall temperature gradient, it is common experience that they often have *fronts*—bands of strong near-surface temperature gradient—embedded within them. These are usually marked on surface synoptic charts by thick lines barbed with symbols: triangles for cold fronts, semicircles for warm fronts, and both for occluded fronts (*q.v.*). The barbs are put on the side towards which the front is moving. We saw some examples in Fig. 7.7; more typical structures are evident 12 hrs later, shown in Fig. 7.8. Notice how the surface pressure



field is distorted by the presence of the fronts: there is usually strong curvature in the surface isobars (contours of constant pressure) in conjunction with a pressure trough at the frontal position.

### 7.5.1 Frontogenesis

How do fronts form in the first place? We know that baroclinic systems must develop where there is a temperature gradient (and therefore available potential energy), but why do tight gradients form? the answer is that gradient-tightening is inevitable, in the presence of flow *deformation*. Since $w = 0$ at the surface, we can think about the effects of the flow on surface temperature purely in terms of horizontal advection (neglecting non-adiabatic effects, for now). In general, we can characterize the horizontal shear in flow by the four terms

$$\begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix}.$$

Alternatively, we can express these by the four independent linear combinations

$$\begin{pmatrix} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} & \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \\ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} & \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \end{pmatrix}.$$

We have already encountered two of these combinations: the divergence $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ (which is zero for geostrophic flow), and the vorticity $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$. the other two terms are expressions of the deformation.

Consider what the flow does to a material box of surface air, of dimension $\delta x \times \delta y$. First, consider the evolution of the area of the box. The area evolves

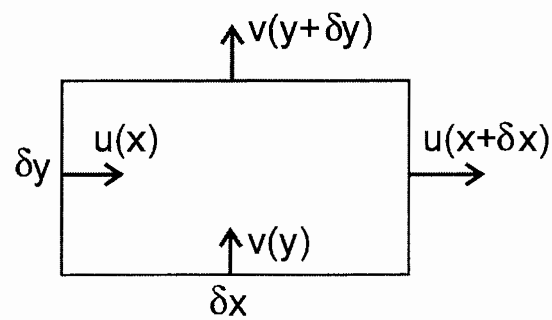


Figure 7.9:

as (see Fig. 7.9)

$$\begin{aligned} \frac{d}{dt}(\delta x\, \delta y) &= \delta x\ \ (v(x, y+\delta y) - v(x,y)) + \delta y\ \ (u(x+\delta x, y) - u(x,y)) \\ &= \delta x\, \delta y\ \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\ . \end{aligned}$$

So the area is preserved in a nondivergent flow.

We know that the vorticity does—it rotates the fluid elements. This leaves the deformation. Consider the "pure deformation" flow described by the streamfunction $\psi = -Kxy$, where $K$ is a constant, shown in Fig. 7.10. Such a flow is nondivergent, and irrotational (zero vorticity). However, it

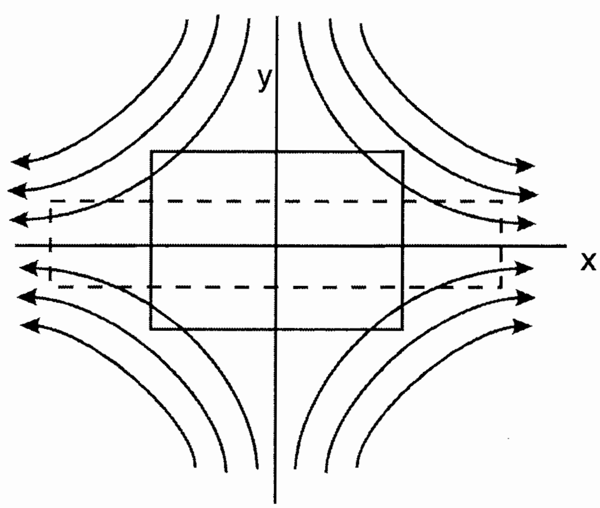


Figure 7.10:

has deformation, since

$$\begin{aligned} \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = -2\frac{\partial^2 \psi}{\partial x \partial y} = 2K\ ; \\ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} &= \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2} = 0\ . \end{aligned}$$

Consider (Fig. 7.10) what happens to a material element such as the solid rectangle on the figure. With the given flow configuration, the box will be stretched in the $x$-direction (the *axis of dilation*), as shown; since area is conserved, it must simultaneously contract in the $y$-direction (the *axis of contraction*). Thus, in this flow, any temperature gradient in the $y$-direction will be intensified by the flow, thus forming a temperature front.

Now consider a developing baroclinic wave. If the temperature gradient

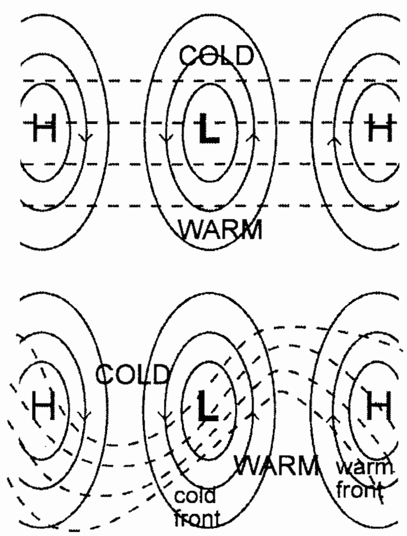


Figure 7.11:

is initially N-S (top of Fig. 7.11), it will be deformed (by the deformation between low and high) and twisted (by the cyclonic vorticity around the low), much as shown in the bottom of Fig. 7.11. Thus, it will form a warm front ahead of the low, and a trailing cold front behind.

### 7.5.2 Frontal evolution

The "textbook" picture of frontal evolution is as depicted in Fig. 7.12, The fronts form a "warm sector", usually to the south of the low pressure center (in the northern hemisphere). The warm sector moves around the storm a little, and contracts; sometimes the fronts merge near the storm center to form an "occlusion." A secondary low pressure center may from at the point of occlusion.

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### 7.5.3 Frontal structure and weather

Fronts may be most intense near the ground, but they do extend vertically. As shown in Fig. 7.13, they slope with height, with the warm air overlying

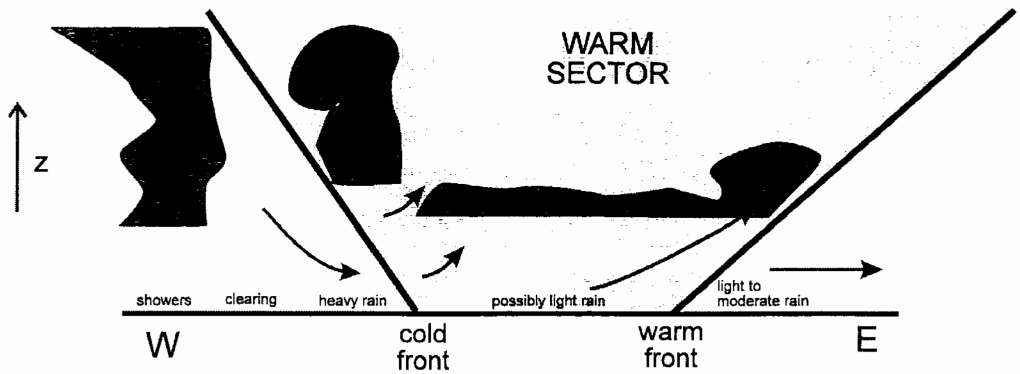


Figure 7.13: A schematic cross-section through the warm sector of a mid-latitude cyclone, running approximately west-east. Frontal motion is to the east; arrows denote air movement.

the denser cold air. At the leading warm front, the warm air rides up over the front. If the air is moist, this will produce clouds and, if the system is energetic enough, rain (or perhaps snow); note that the precipitation at the front, though it may be formed in the warm air, will fall through cold air—the precipitation is formed aloft, and so will be ahead of the surface front. Within the warm sector, there is weak upwelling, so this sector will often be completely cloud covered and there may be extensive rain or drizzle. At the cold front, the cold air undercuts the warm air, pushing the latter upward. At the front itself, there may be heavy rain or snow; some time later, there is a clearing in the subsiding air behind the front. Later still, convection may occur, which in some systems can be intense: cold air is moving over ground that is warm, following passage of the warm sector, and so the temperature structure is often convectively unstable. Immediately behind the front, convection may be suppressed by the subsidence. Once this abates, convection may set in. (Intense thunderstorms often follow the passage of summertime cold fronts.)

At the apex of the warm sector, the fronts may occlude. Often, this takes the form of the cold/warm front intersection leaving the ground, as the warm sector gets squeezed aloft. Then air at the ground is everywhere cold, as in Fig. 7.14. The warm air slides up the occlusion, from the warm

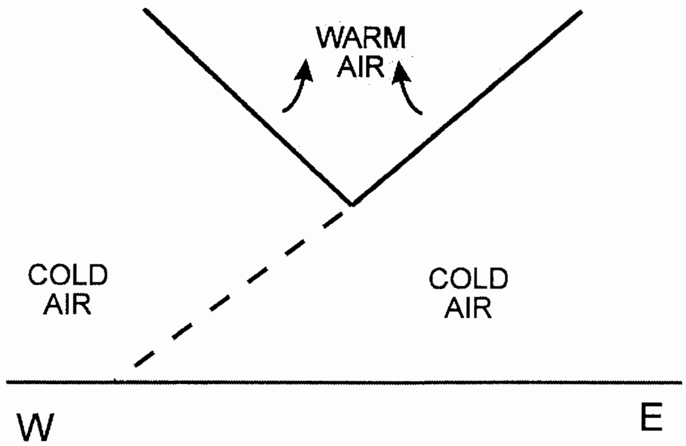


Figure 7.14: Cross-section (nominally W-E) through an occlusion.

sector. Precipitation is frequent in such situations; in winter it is frequently snow (in New England, this is the classic snowstorm), as the precipitation falls through a deep layer of cold air.

## 7.6 Climatology of synoptic systems: storm tracks

Synoptic-scale storm systems are not uniformly distributed over the globe; they concentrate in middle latitudes of the winter hemisphere and, even there, are more common in some longitudes than others. Fig. 7.15 shows the average distribution of eddy kinetic energy density $\frac{1}{2}\left(u'^2 + v'^2\right)$ and r.m.s. geopotential variance $\sqrt{z'^2}$ for northern winter. There are two main "storm tracks" where these quantities are large: across the N Pacific and N Atlantic Oceans.

The reason for this structure is the planetary scale structure of the background state. Storms tend to be generated in the regions of strongest *baroclinicity*—temperature gradient—off the E coasts of Asia and North America. Once formed, the storm motion is steered by the planetary scale flow which, as we saw in Fig. 6.3, is southwesterly (northeastward) across these oceans.

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Thus, the synoptic scale systems are controlled by the planetary wave flow; in turn, the synoptic systems influence the planetary scale flow also. Eventually, the storms dissipate, though Pacific storms may propagate across the Atlantic and beyond before they die.

The situation in southern hemisphere winter is shown in Fig. 7.16; the main southern storm track extends across the southern Atlantic and Indian oceans.

## 7.7 Further Reading

Holton, J.R., "An Introduction to Dynamic Meteorology", Academic Press, 1979.

James, I.N., "Introduction to Circulating Atmospheres", Cambridge University Press, 1994.

Wallace, J.M., and P.V. Hobbs, "Atmospheric Science: An Introductory Survey", Academic Press, 1977.

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