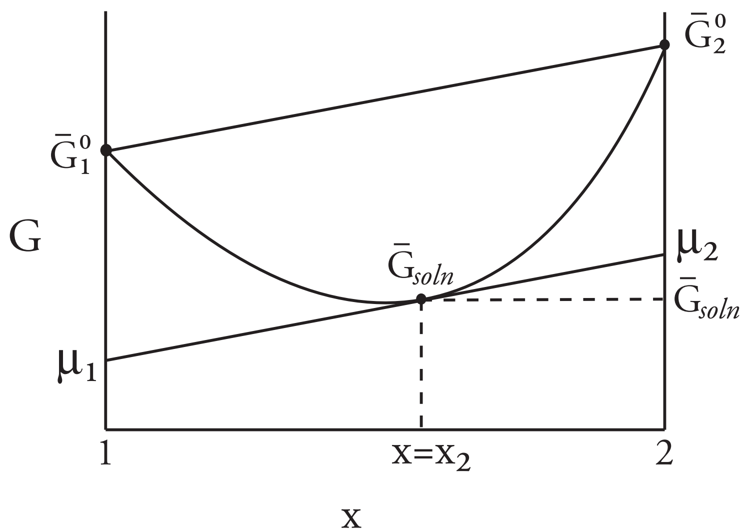
Free Energy of a Solution

We can write the free energy of a solution as follows:
2 components 1&2 $x =$ mole fraction.

$$\bar{G}_{soln} = \underbrace{x_1\bar{G}_1^0 + x_2\bar{G}_2^0}_{G_{\text{mechanical mixing}}} + \underbrace{nRT(x_1 \ln x_1 + x_2 \ln x_2)}_{-T\Delta S \text{ mix}} \tag{1}$$

The definition of the chemical potential:

$$\left.\frac{\partial G}{\partial x_i}\right|_{P,T,n_j \neq n_i} = \mu_i$$



$$\frac{\partial \bar{G}_{soln}}{\partial x_1} = \bar{G}_1^0 - \bar{G}_2^0 + nRT[\ln x_1 - \ln(1 - x_1)] \tag{2}$$

$$\mu_2 = \bar{G}_{soln} + (1 - x_2)\frac{\partial \bar{G}_{soln}}{\partial x_2} \tag{3}$$

$$\mu_1 = G_{soln} + x_2\frac{\partial G_{soln}}{\partial x_2} \tag{4}$$

then:

$$\mu_1 = G_1^0 + nRT \ln x_1$$

$$\mu_2 = G_2^0 + nRT \ln x_2$$

Study Question

1. From:

$$dG \leq -SdT + VdP + \sum_i \mu_i dx_i$$

show that:

$$\frac{\partial G}{\partial x_2} = \mu_2 - \mu_1$$

2. Using the definition for $\bar{G}_{soln}$, an expression for $\frac{\partial \bar{G}_{soln}}{\partial x_2}$, and $\mu_2 = \bar{G}_{soln} + (1-x)\frac{\partial \bar{G}_{soln}}{\partial x}$, show that:

$$\mu_2 = \mu_2^0 + nRT \ln x_2$$