

## 12.520 Lecture Notes #4

Assertion: most of the stress tensor in the Earth is close to "lithostatic,"

$$\tau_{ij} \sim -\rho gd \delta_{ij},$$

where  $\rho$  is the average density of the overburden,  $g$  is gravitational acceleration, and  $d$  is the depth of the point under consideration.

Consider the following table of the lithostatic pressure at various points of interest:

	depth (km)	Pa	bars
ocean ridge crest	2.5	25 MPa	250
abyssal plain	4	40 MPa	400
deep sea trench	12	120 MPa	1.2 kbar
base of crust	30	1 GPa	10 kbar
transition zone	600	20 GPa	200 kbar
Core-mantle bndry	2900	140 GPa	1.4 Mbar
Center of Earth	6400	360 GPa	3.6 Mbar

Assertion: Typical "tectonic" stresses have magnitudes in the range 0.3 - 300 MPa (3-3,000 bars).

Conclusion: In considering tectonic problems, it is usually useful to subtract some isotropic reference state of stress. There are 2 common choices - the lithostatic stress, which leaves the nonlithostatic stress tensor, and the pressure field, which leaves the deviatoric stress field. But since we need to know the actual stress tensor in order to calculate the pressure, which requires knowing the solution to the problem in advance, it is most common to use the lithostatic stress field as the reference.

In order to place these comments in context, let's look at the situation described in T&S problem 2-6, where a continent is in isostatic equilibrium with an ocean. To address the possible state of (nonlithostatic) stress in the region, we will follow an iterative approach. The "algorithm" can go as follows:

- 1) Assuming lithostatic stress, calculate the horizontal normal stress as a function of depth beneath both the continental and the oceanic structures.
- 2) Evaluate the "reasonableness" of this assumption by making a free body diagram by isolating a (thin) vertical "pill box" of material at the continent-ocean boundary and calculating the total forces acting on its sides, given this assumption.

This can be done by integrating the normal traction as a function of depth to determine the total normal force (per unit length into the picture) acting on each of the two vertical faces of the pill box:

$$F_x/\text{length} = \int_{\text{surface}}^{\text{Moho}} \sigma_n dy$$

The normal traction exerted by the continent on the pill box is everywhere greater than the normal traction exerted by the oceanic structure on the pill box. The total force on the pill box is just the area shaded with horizontal hatchure in the figure below.

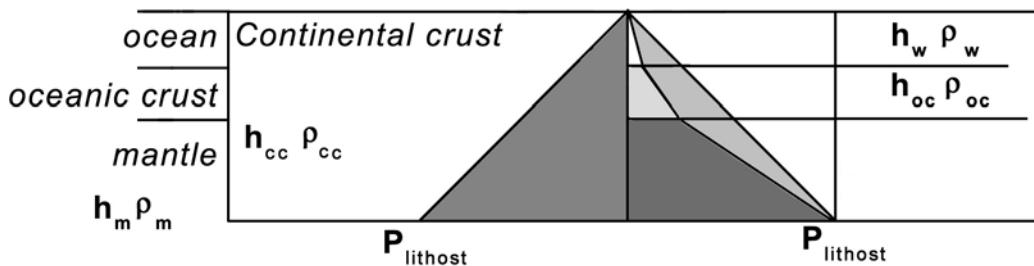


Fig. 4\_1

- 4) Since our assumptions have led to an unbalanced force on the pillbox that would lead to an infinite acceleration as the thickness of the pill box shrinks, something must be wrong with our assumption. So we need to modify our model to decrease the magnitude of the normal traction exerted by the continent, increase the normal traction exerted by the oceanic structure, or both.
- 5a) If we "fix" our problem in the continent, we require a nonlithostatic horizontal extension.
- 5b) If we "fix" our problem in the oceanic region, we require a nonlithostatic horizontal compression.
- 5c) Of course, another solution would be to have horizontal compression in the continent, with even more horizontal compression in the oceanic area.

**Note: Isostasy and lithostatic stress cannot exist simultaneously!** Areas of thickened crust tend to have a state of stress that is more extensional than nearby regions of thin crust.

Examples: Tibet, Southern California Transverse Ranges.

Sources of tectonic stress are of great interest to geodynamics, particularly if variations in stress can be interpreted in terms of interesting processes. Recall, from equilibrium,

$$\tau_{ij,j} + \rho f_i = 0$$

If we can ignore body forces, then, for example,

$$\frac{\partial \tau_{xx}}{\partial x} = -\frac{\partial \tau_{xz}}{\partial z}$$

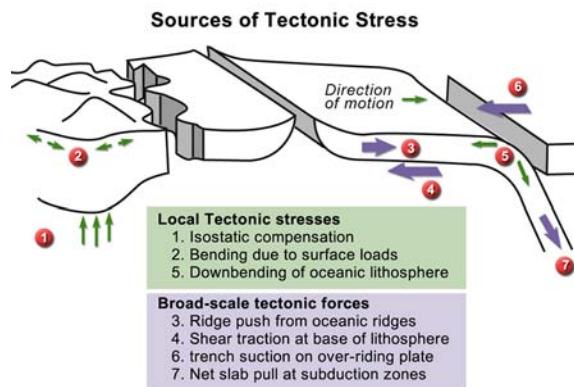


Fig. 4\_2

Example:

Stress transition near back-arc basin

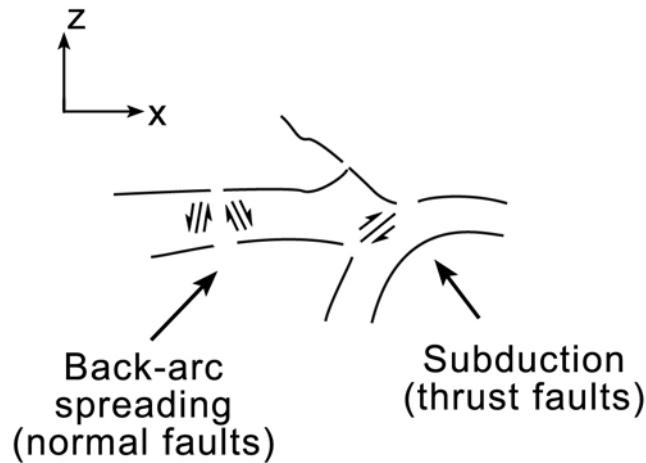


Fig. 4\_3

## Measurements:

- “Flat jack” – null measurement

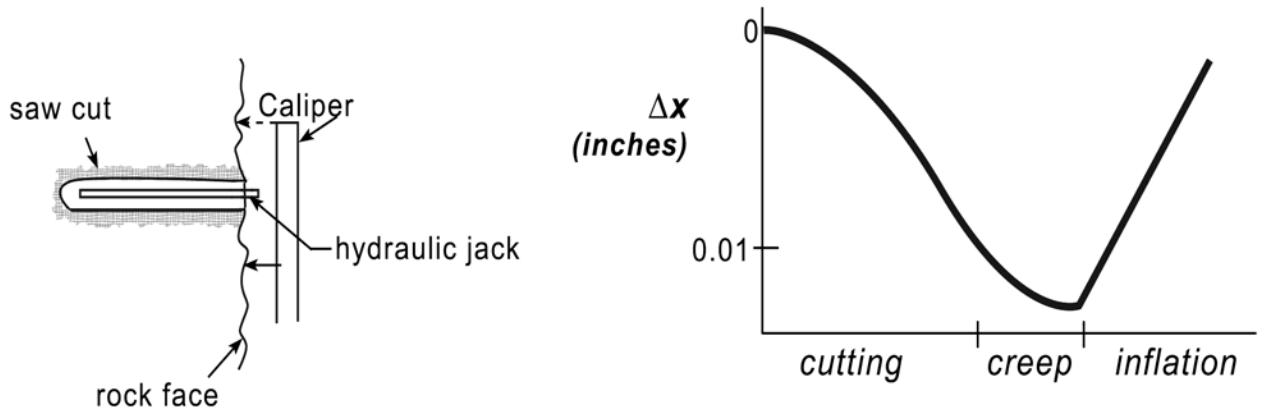


Fig. 4\_4

Given  $T_n$  across cut

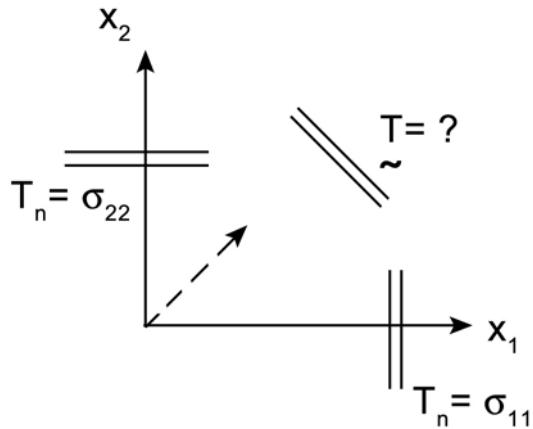


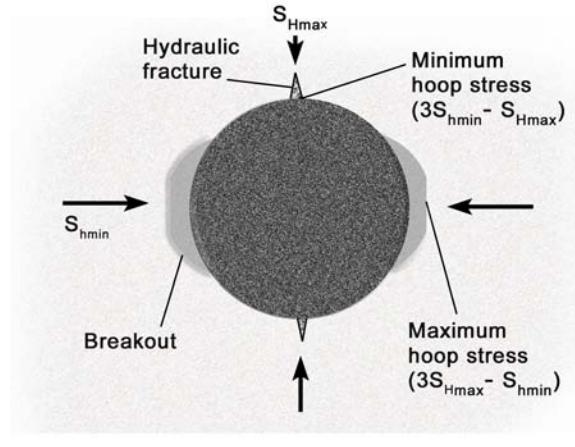
Fig. 4\_5

Related techniques – overcoring  
Hydrofracture

Problems – Near surface heterogeneity  
Difficult to do remotely  
Only gives present-day stress

Other techniques – direction only  
Cheap or free!

- Borehole breakout – relies on stress concentration around drill holes; spalling of borehole walls



Stress concentration around a vertical circular hole in an elastic half-space subjected to unequal far-field horizontal stresses,  $S_{h\text{min}}$  and  $S_{h\text{max}}$ . As indicated, breakouts and hydraulic fractures form as a result of these stress concentrations.

Fig. 4\_6

### Changes in stress direction

Example – subduction zone

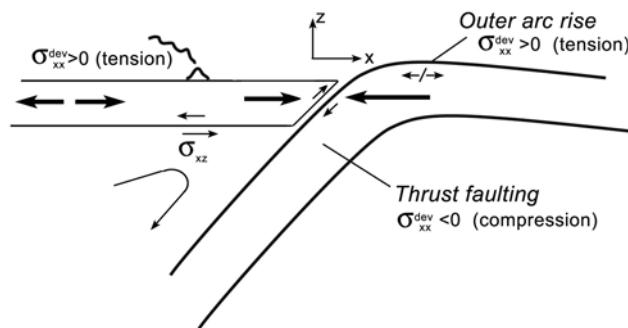


Fig. 4\_7

Fore-arc: Compression  $\sigma_{xx}^{dev} < 0$

Back-arc: Extension  $\sigma_{xx}^{dev} > 0$

$\Rightarrow$  mantle flow “required”

$$\text{Equilibrium: } \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} = 0$$

{Outer rise – extension near surface due to bending (discussed later)}

Other examples of change in stress orientation:

US – Mid continent ~ E-W Compression

Basin and range ~ E-W extension

California – rotation of dev. Stress near faults

Earthquake focal mechanisms

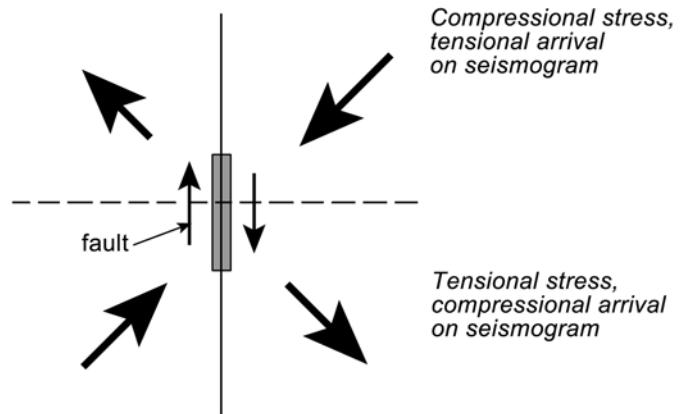


Fig. 4\_8

Types of faults

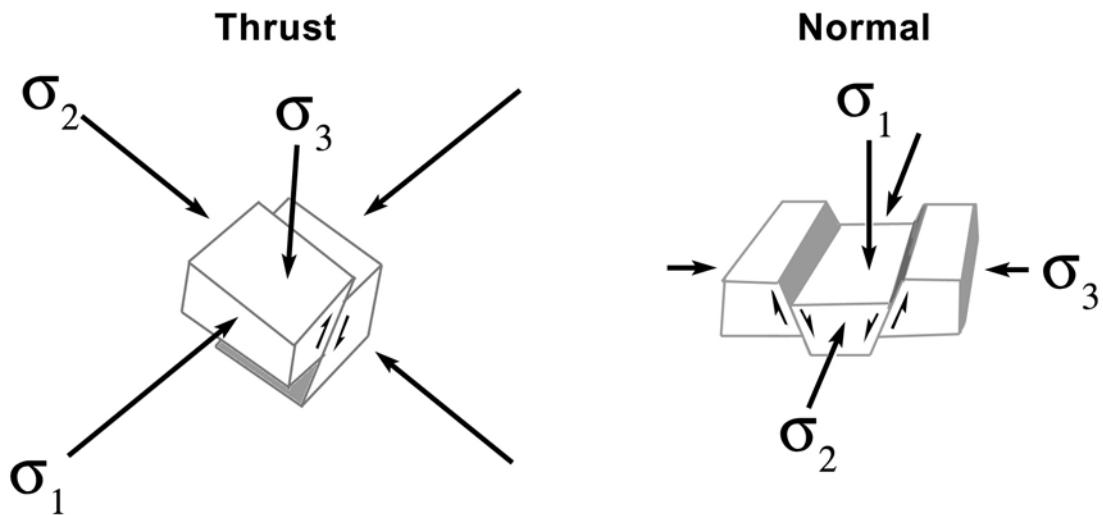


Fig. 4\_9

Strike slip  
(map view)  
 $\sigma_2$  vertical

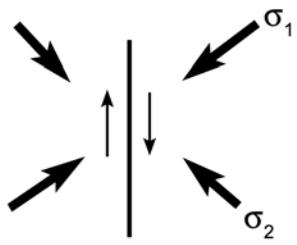


Fig. 4\_10

Note: Angle of  $\sigma_1$  to fault plane depends on fault properties.