Class 3: Refraction Traveltime Interpretation
Wed, Sept 16, 2009

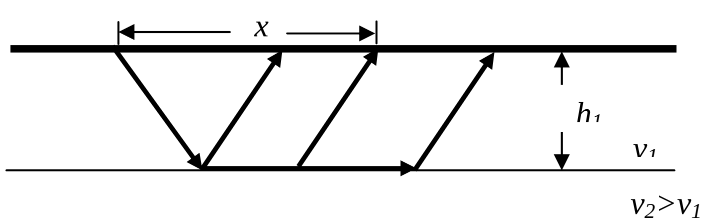
- Analytical refraction solutions
- Detailed approach in delay-time method
- Generalized Linear Inversion (GLI) for layer model inversions
- Review of commercial software solutions
- Computer exercise – process synthetic and real data

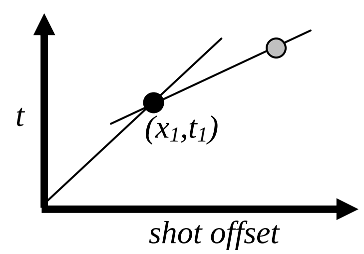
This class introduces the conventional refraction traveltime methods for near-surface structure interpretation. These approaches produce layered models, appropriate for relatively simple velocity structures. Two influential approaches include delay-time method and generalized liner inversion method. We invited two distinguished geophysicists in these areas to give presentations, Chuck Diggins, Chief Geophysicist of FusionGeo, and Dan Hampson, President of Hampson and Russell. However, both of them are traveling this week. They will join us on Sept 28, 2009.

Analytical Refraction Solutions

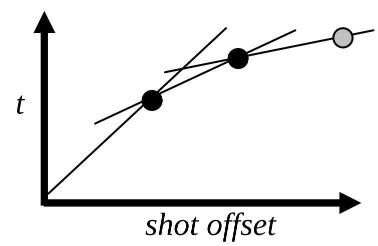
Assuming 1D velocity structures with multiple layers and velocity increasing with depth, we can derive an exact analytical solution from refraction traveltimes.

Two-Layer Model (One layer over half a space):





$$t_1 = x_1/v_2 + 2h_1(v_2^2 - v_1^2)^{1/2}/(v_2 v_1) \qquad (1)$$



Three-Layer Model (Two layers over half a space):

$$t_2 = x_2/v_3 + 2h_1(v_3^2 - v_1^2)^{1/2}/(v_3 v_1) + 2h_2(v_3^2 - v_2^2)^{1/2}/(v_3 v_2) \qquad (2)$$

N-Layer Model

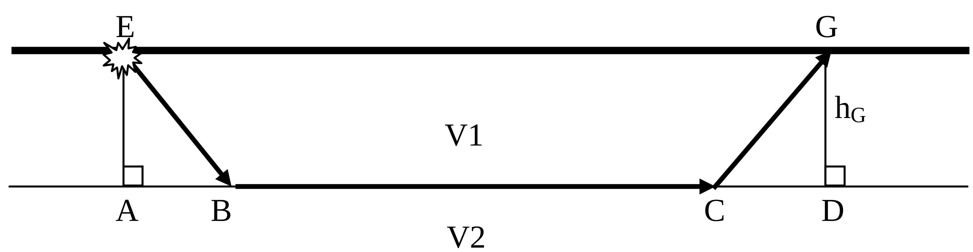
$$t_{n-1} = x_{n-1}/v_n + (2/v_n)\Sigma h_i(v_n^2 - v_i^2)^{1/2}/v_i \qquad (3)$$

$$t_{n-1} = a_0\, x_{n-1} + a_1 h_1 + a_2 h_2 + a_3 h_3 + \ldots \qquad (3a)$$

Notes:

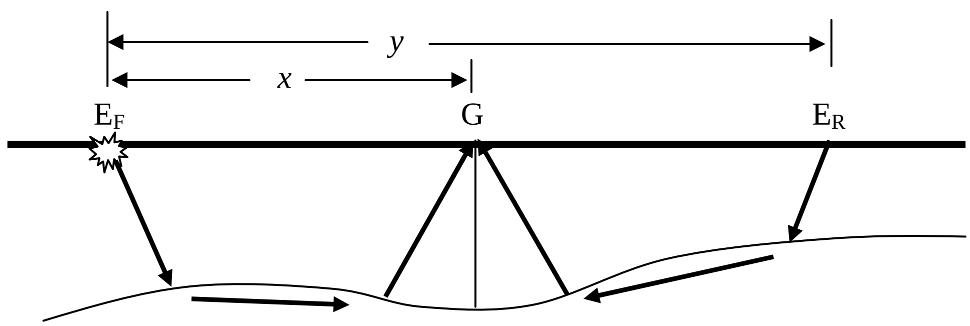
- Other analytical solutions – multiple dipping interfaces
- All analytical solutions _ assume velocity increasing with depth
- Hidden layers – refraction methods fail
- Any other structures – no analytical solutions, need to be derived otherwise
- All approaches are under some assumptions

Delay-Time Method
Map irregular refractor interfaces



Total delay time for source and receiver: $T_{EG} = T_E + T_G$ (1)

Delay-time at G: $\mathbf{T_G} = CG/v_1 – CD/v_2 = h_G ((v_2^2-v_1^2)^{1/2} /(v_1v_2))$ (2)

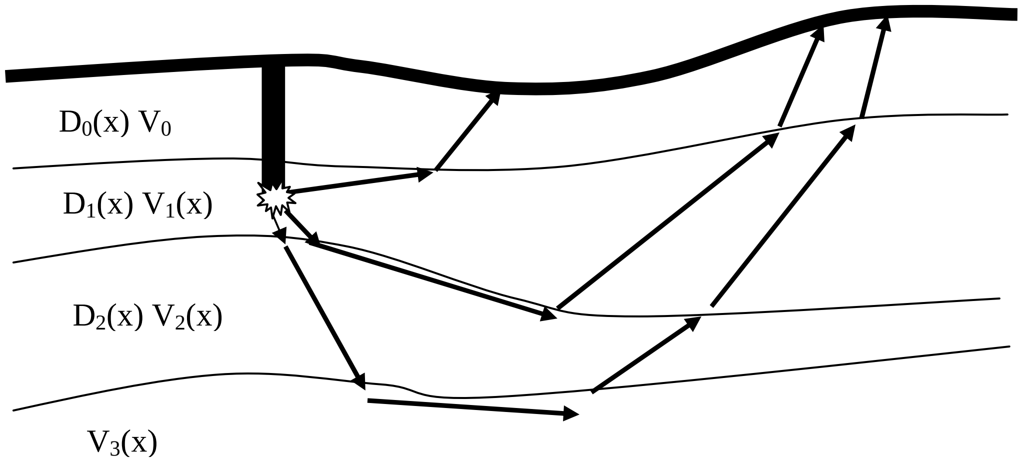


Derive reliable v_2 when the interface varies:

$t_{EFG} – t_{ERG} = T_{EF} – T_{ER} + 2x/v_2 – y/v_2$ (3)

Questions: what are the assumptions behind the delay-time method?

Generalized Linear Inversion (GLI) for Layer Models
By Hampson and Russell (1984)



Objective: given traveltimes observed at the surface, invert layer depths $D_i(x)$, and velocity $V_i(x)$

Model: $M=(m_1, m_2, m_3 \ldots, m_m)$ (thickness and **slowness**)
Data: $T=(t_1, t_2, t_3, \ldots, t_n)$ (direct-wave and refraction arrivals)

Physics: $t_i=A_j(m_1, m_2, m_3 \ldots, m_m), i=1,2,3, \ldots n.$
Given a velocity model and geometry, generate traveltimes of rays

Inverse Problem:

$$B\Delta M=\Delta T$$
$$B_{ij}=\partial t_i/\partial m_j$$

Objective Function: $\psi = || T - A(m)||^2$

Model Update: $\Delta M=(B^T B)^{-1} B^T \Delta T$
$M=M+\Delta M$

A more sophisticated inversion scheme:
Objective Function: $\psi = \|T - A(m)\|^2 + \tau \|L(m)\|^2$
Solving an inverse problem:

$$(B^T B + \tau L^T L)\Delta M=B^T \Delta T - \tau L^T L(M)$$

Commercial Software Packages:

Delay-Time Solutions:

Fathom:	Green Mountain Geophysics (ION), Denver, CO
Seismic Studio:	FusionGeo, Houston, Texas
TomoPlus:	GeoTomo, Houston, Texas

Generalized Linear Inversion:

GLI3D:	Hampson-Russell, Calgary, Canada