12.800: Problem set 4

1) In the last problem set you derived the momentum equation up to the form

$$\rho\frac{D\mathbf{u}}{Dt}=\rho g+\nabla\cdot\boldsymbol{\tau}.$$

Now show that

$$\nabla\cdot\boldsymbol{\tau}=-\nabla p+\mu\nabla^2\mathbf{u}+\frac{\mu}{3}\nabla(\nabla\cdot\mathbf{u})$$

by following the derivation done in class and carefully describing the steps taken and assumptions made.

2) In class we derived the heat equation by conserving total energy and then removing the mechanical energy component.

Starting from $\rho\frac{De}{Dt}=-p(\nabla\cdot\underline{u})+\phi-\nabla\cdot\underline{q}$, produce the relationship

$$\rho C_p\frac{DT}{Dt}=\phi-\nabla\cdot\underline{q}$$

without resorting to the relationship $de=C_v dT$.

Hints: Use $de=-pd\alpha+Td\eta$ where $\alpha=\frac{1}{\rho}$, and η is the entropy per unit mass (the intensive version of entropy). You'll need to know how C_p is related to entropy per mass, and you'll have to make assumptions about certain terms being small.