12.800: F04 Problem set 9

1) Assume that the atmospheric Ekman layer over the earth's surface at a latitude of 45 degrees north can be approximated by an eddy viscosity of $\nu_v = 10\ \text{m}^2/\text{s}$. If the geostrophic velocity above the Ekman layer is 10 m/s, what is the Ekman transport across isobars? [KC04: 14.3]

2) Between 15 degrees north and 45 degrees north, the winds over the North Pacific consist mostly of the easterly trades (15 degrees north to 30 degrees north) and the westerlies (30 degrees north to 45 degrees north). An adequate representation of the zonal wind stress is

$$\tau_x = \tau_0 \sin\left(\frac{\pi y}{2L}\right), \quad \tau_y = 0, \quad -L \le y \le L,$$

where $\tau_0 = 0.15\ \text{N/m}^2$ is the maximum wind stress and $L = 1670$ km. Taking $\rho_0 = 1028\ \text{kg/m}^3$ and the value of the Coriolis parameter corresponding to 30 degrees north, calculate the Ekman pumping. Which way is it directed (up or down)? Calculate the vertical volume flux over the entire 15 degree to 45 degree strip of the North Pacific (width = 8700 km). Express you answer in sverdrup units (1 sverdrup = 1 Sv = $10^6\ \text{m}^3/\text{s}$). [CR94: 5.6]

3) In class we derived

$$\frac{D\omega_z}{Dt} + \beta v = 0 \qquad (1)$$

using Kelvin's Circulation Theorem for a rotating, inviscid, barotropic, fixed depth fluid. Previously, we derived a vorticity equation for a rotating fluid given by

$$\frac{D\boldsymbol{\omega}}{Dt} = ((\boldsymbol{\omega} + 2\boldsymbol{\Omega}) \cdot \nabla)\mathbf{u} + \frac{1}{\rho^2}\nabla\rho \times \nabla p + \nu\nabla^2\boldsymbol{\omega}, \qquad (2)$$

and if the fluid is assumed to be inviscid, barotropic, and fixed depth, this equation appears to reduce to

$$\frac{D\boldsymbol{\omega}}{Dt} = 0. \qquad (3)$$

Explain why equation (1) is different from equation (3). Which is correct?

4) Derive expressions for the dispersion relation, phase speed, and group velocity of the Rossby wave solutions of equation (1) above. Include a plot of contours of ω / β as a function of zonal and meridional wave numbers.