1. Consider the following linearized equations in 1 and 3 dimensions and impose a wave solution as shown:

Linearized Equation	Plane wave
$\phi_t + c\phi_x = 0$	$e^{ikx-i\omega t}$
$\phi_{tt} - c^2\phi_{xx} = 0$	$e^{ikx-i\omega t}$
$\phi_t + \vec{c}.\nabla\phi = 0$	$e^{i\vec{k}\cdot\vec{x}-i\omega t}$
$\phi_{tt} - c^2\nabla^2\phi = 0$	$e^{i\vec{k}\cdot\vec{x}-i\omega t}$
$\nabla^2\phi_t + \beta\phi_x = 0$	$e^{i\vec{k}\cdot\vec{x}-i\omega t}$

Find the dispersion relation for each of them and the phase speed (or three phase speed in 3 dimensions)

2. Suppose a wave is found that has the form

$$\phi = \mathrm{A}e^{i\theta}$$

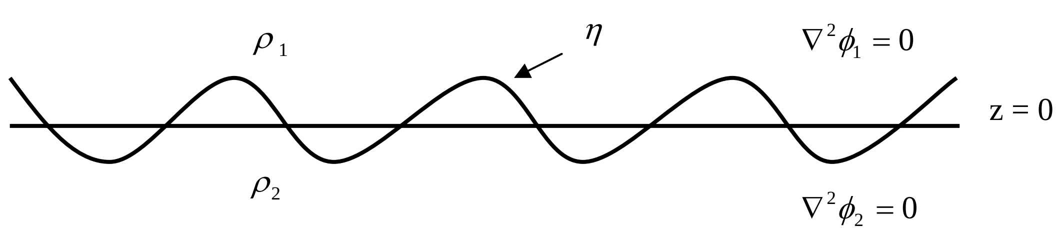
where

$$\theta = -\alpha\, t^2 / x$$

a) If the wave can be thought of as slowly varying, what are its frequency, wavenumber and dispersion relation?

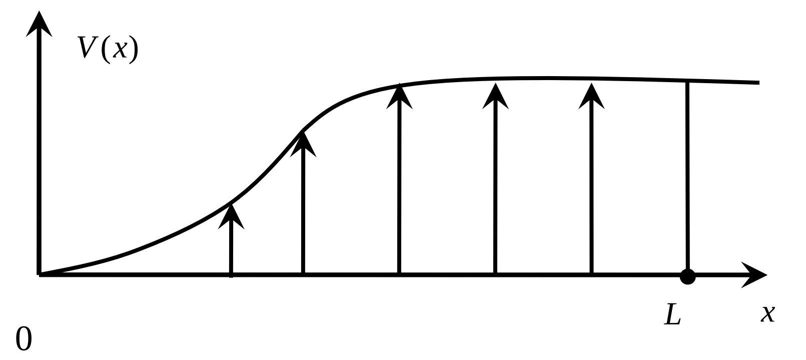
b) Now that you have ω and k, when will the slowly varying assumption be valid?

3. Consider the interface between two semi-infinite fluids of different densities:



Imposing wave solutions for (ϕ_1, ϕ_2, η) that obey the deep water equations derive the boundary conditions that must be satisfied at $z = 0$ and the dispersion relation. What type of wave have you obtained?

4. Consider a deep water wave impinging on a current $V(x)$ of the following shape:

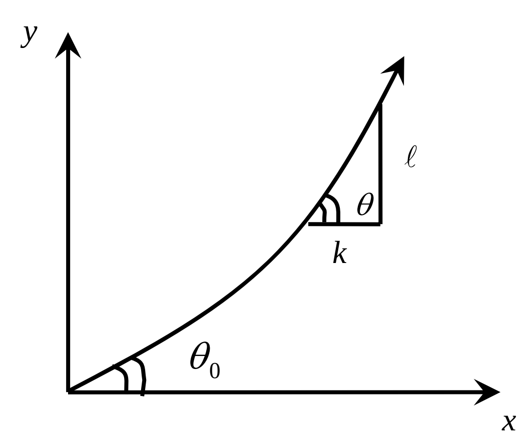


a) What is the dispersion relation?

b) Derive the ray equations for (ω, k, ℓ) and discuss their implications.

c) A wave packet starts its motion with initial conditions (ℓ_0, k_0, ω_0) where $V(x) \cong 0$ and impinges on the current. What is $k(x)$? Sketch the variation of the wave from $x = 0$ to $x = L$.

d) If the wave ray moves as in the sketch:



What is $\sin\theta(x)$? What is the ratio $\dfrac{\sin\theta(x)}{\sin\theta_0}$?

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