1. For surface gravity waves, to calculate the trajectories of fluid elements in the plane wave, let (ξ, ζ) be the (x, z) displacements of the fluid element around some original position (x_0, z_0), so that at zero order

$$\begin{cases} \dfrac{d\xi}{dt} = u\,(x_0, z_0, t) \\[2ex] \dfrac{d\zeta}{dt} = w\,(x_0, z_0, t) \end{cases}$$

Find the analytical shape of the trajectory. What happens at the bottom $z = -D$? What is the limiting shape of the trajectories when $z \to -\infty$?

2. Evaluate the three components of relative vorticity:
$(v_x - u_y)$; $(w_y - v_z)$; $(u_z - w_x)$ for

   a) surface gravity waves over depth D

   b) Internal waves, $w = w_0 \cos(kx + ly + mz - \omega t)$
   (Express every field function in terms of w using the equations of motion.)
   Compare a) and b). What happens if $N^2 = 0$?

3. a) Derive the energy equation for internal gravity waves. You will need to use the adiabatic energy equation to obtain the expression of the potential energy in conservation form.

   b) Assuming a solution $w = w_0 \cos(kx + ly + mz - \omega t)$, show that the kinetic and potential energies averaged over one wavelength are equal, i.e. there is equipartition of energy. (Express every field function in terms of w using the equations of motion.)
   Express the average $\overline{E}$ in terms of the angles (θ, ϕ) defined in class where the "—" denotes the average over one wavelength.

c) Evaluate explicitly the group velocity $\vec{c}_g = \left(\frac{\partial \omega}{\partial k}, \frac{\partial \omega}{\partial \ell}, \frac{\partial \omega}{\theta m} \right)$ and express its components in terms of (θ, ϕ).

d) Evaluate the three components of the energy flux vector $\vec{F}$, i.e. (pu, pv, pw) as $\vec{F} = p\vec{u}$.

Show that $\overline{F_x} = \overline{E}\, c_{gx}$; $\overline{F_y} = \overline{E}\, c_{gy}$; $\overline{F_z} = \overline{E}\, c_{gz}$ where $\overline{E} = \left(\frac{1}{2} \right) p_0 \left(\frac{w_0}{\cos\theta} \right)^2$ is the average energy for wavelength, so that $\vec{F} = \overline{E}\, \vec{c}_g$.