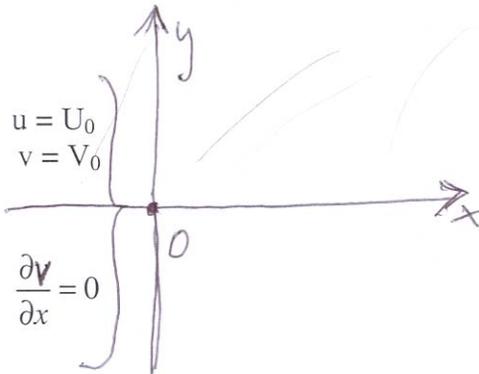


1. A steady flow with components

- i. $u=U_0=\text{constant} > 0$
- ii. $v=V_0=\text{constant} > 0$



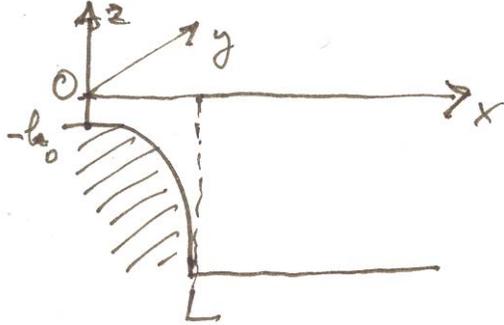
Is given at $x=0$ on the semi-infinite β -plane $x > 0$ as shown in the sketch, with the given boundary conditions as $x=0$. The ocean is homogeneous, frictionless and with constant depth. H_0 . Find the steady state solution for $x > 0$ and discuss its physical meaning. (Note that also $\frac{\partial v}{\partial x} = 0$ at $x=0$).

2. We are on the equatorial β -plane

- a. A Poincaré wave-packet moves northward starting from an initial condition with (ω_0, k_0, l_0) - what is its turning latitude y_T and what defines it?
 - b. Repeat the calculation for an equatorial Rossby wave packet.
3. Consider depth integrated barotropic flow on the f -plane and on the shelf:

$$h(x) = -h_0 e^{\lambda x} \text{ for } x \leq L$$

$$h(x) = -h_0 e^{\lambda L} \text{ for } x \geq L$$



The flow is forced by wind stress and bottom friction and obeys the following equations

$$U_t - fV = -h \frac{\partial p}{\partial x} - \frac{rU}{h}$$

$$V_t - fU = -h \frac{\partial p}{\partial y} + \tau - \frac{rV}{h}$$

$$U_x + V_y = 0$$

r is the bottom friction coefficient. Consider the case in which the flow is uniform in y ,

$$\frac{\partial}{\partial y} = 0$$

The wind stress has the simple shape $\tau = \tau_0 e^{i\omega t}$

Assign the appropriate boundary conditions. Solve for the alongshore transport V . Explain how the response changes according to the importance of bottom friction, i.e. for r very large or very small.

4. Consider a layer of fluid with a buoyancy frequency N (constant). The fluid is flowing in the positive x - direction with constant velocity U . The base of the fluid is rippled such that

$$h = h_0 \cos kx$$

Where h is the (small) departure of the bottom of the fluid from a flat surface. *The upper surface of the fluid is level and rigid at a distance D from the bottom.*

- Find the steady solution for the flow (it is nonrotating).
- Discuss whether resonance can occur and interpret your results.
- Calculate the drag on the rippled boundary and discuss the results in form of the net energy radiation.

5. Consider an infinite channel on the zonal β -plane with homogeneous fluid, a rigid lid and constant depth. The channel extends from $y=-L_S$ to $y=+L_N$. For $y < 0$ the beta parameter is β_S and for $y > 0$ beta in β_N . The wave motions obey the barotropic vorticity equation on the β -plane. Assign the appropriate boundary conditions at $y=L_S, L_N$ and $y=0$.

Find the solutions and the condition for them to be wavelike in both halves of the channel. Are there other possible types of solutions? Please discuss.

6. Suppose we model the southern boundary Y_S of the Gulf Stream as a rippling surface propagating eastward. We prescribe that boundary at $y = 0$

$$\psi(x, 0, t) = \text{Re} e^{ik_o(x-ct)}$$

Now consider the oceanic region south of that boundary (i.e., $y \leq 0, y \rightarrow \infty$). Describe the resulting possible wave radiation in the region $y < 0$. Consider both positive and negative values of c .

7. Consider a two-layer fluid, with layer depths D_1 and D_2 , obeying the dimensionless linearized equations of quasi-geostrophic dynamics:

$$\frac{\partial}{\partial t} \left[\frac{\partial^2 \psi_1}{\partial x^2} + \frac{\partial^2 \psi_1}{\partial y^2} - F_1(\psi_1 - \psi_2) \right] + \beta \frac{\partial \psi_1}{\partial x} = 0$$

$$\frac{\partial}{\partial t} \left[\frac{\partial^2 \psi_2}{\partial x^2} + \frac{\partial^2 \psi_2}{\partial y^2} - F_2(\psi_2 - \psi_1) \right] + \beta \frac{\partial \psi_2}{\partial x} = 0$$

Where $F_1 = f_0^2 L^2 / g' D_1$; $F_2 = f_0^2 L^2 / g' D_2$

With $g' = g \Delta \rho / \rho_0$ the reduced gravity; ψ_1 and ψ_2 the streamfunctions for the upper and lower layers respectively, and the usual meaning for the remaining symbols.

- a. In the infinite (x,y) plane look for plane wave solutions to the above problem. How many solutions are possible in the two-layer model? Find the dispersion relationship and solve for the vertical structure of each wave solution. Draw sketches of the vertical structure.

- b. Consider now the continuously stratified problem:

$$\frac{\partial}{\partial t} \left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial}{\partial z} \left(\frac{1}{S} \frac{\partial \psi}{\partial z} \right) \right] + \beta \frac{\partial \psi}{\partial x} = 0$$

With $S = N^2(z)D^2 / f_0^2 L^2$ the stratification parameter.

How would you solve it, still looking for plane wave solutions? Consider $N^2(z) = \text{constant}$. Write the appropriate dispersion relationship.

- c. Compare the two-layer model to the continuously stratified problem and discuss how the solution of the former relates to the solution of the latter.

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