

3. Equations of motion

For the purposes of this course, we will, for the most part, use the hydrostatic, horizontal equations of motion in local Cartesian coordinates. The full equations may be written (cf. Holton, 1992)

$$\frac{du}{dt} - \frac{uv \tan \varphi}{a} + \frac{uw}{a} = -\alpha \frac{\partial p}{\partial x} + 2\Omega \sin \varphi v - 2\Omega \cos \varphi w + F_x, \quad (3.1)$$

$$\frac{dv}{dt} + \frac{u^2 \tan \varphi}{a} + \frac{vw}{a} = -\alpha \frac{\partial p}{\partial y} - 2\Omega \sin \varphi u + F_y, \quad (3.2)$$

where u and v are the eastward and northward velocity components, and F_x and F_y are the components of frictional acceleration in the eastward and northward directions.

A scale analysis of these momentum equations (cf. Holton, 1992) shows that the centrifugal terms on the left sides of (3.1) and (3.2) are very small compared to the other terms, as is the Coriolis acceleration term involving w in (3.1). Thus, for the purposes of forming simplified equations for developing conceptual understanding of quasi-balanced flows, we shall drop these terms henceforth, and write (3.1) and (3.2) as

$$\frac{du}{dt} = -\alpha \frac{\partial p}{\partial x} + fv + F_x, \quad (3.3)$$

$$\frac{dv}{dt} = -\alpha \frac{\partial p}{\partial y} - fu + F_y, \quad (3.4)$$

where f is the *Coriolis parameter*, defined

$$f \equiv 2\Omega \sin \varphi. \quad (3.5)$$

In the atmosphere, it is usually convenient to write (3.3) and (3.4) in pressure coordinates:

$$\frac{du}{dt} = -\frac{\partial \varphi}{\partial x} + fv + F_x, \quad (3.6)$$

$$\frac{dv}{dt} = -\frac{\partial \varphi}{\partial y} - fu + F_y, \quad (3.7)$$

in which horizontal gradients are understood to be taken at constant pressure.

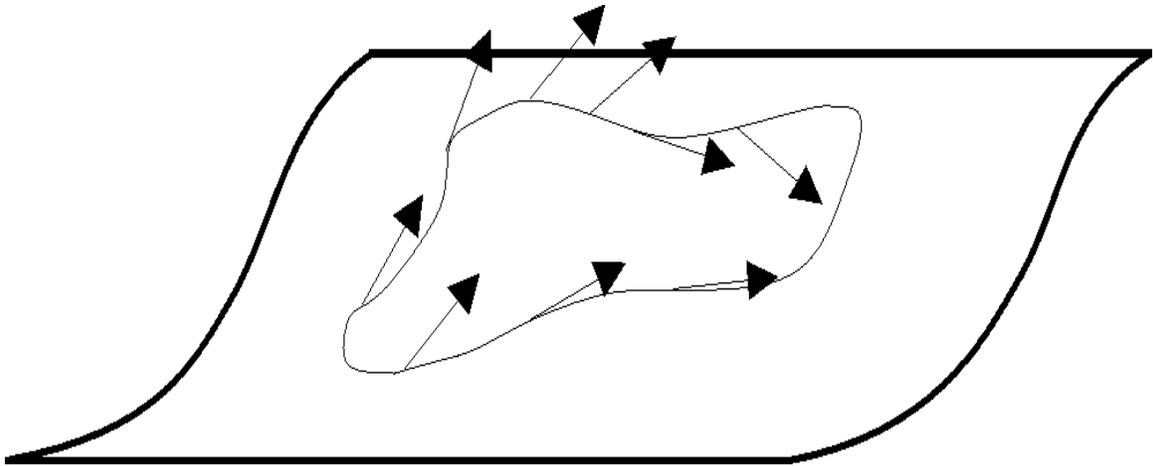


Figure 3.1

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