

12. Eady edge waves

A specific example illustrating the importance of dynamical boundary conditions is a subspecies of Rossby wave, called the *Eady edge wave*. Suppose we have a flow of a quasi-geostrophic fluid in a semi-infinite domain, as illustrated in Figure 12.1a.

This fluid, we shall suppose, has constant pseudo potential vorticity and static stability, \mathcal{S} , and no sources or sinks of q_p , so that

$$q'_p = 0$$

everywhere, for all time. Then (10.5) becomes

$$\frac{1}{f_0} \nabla^2 \varphi + \frac{f_0}{\mathcal{S}} \frac{\partial^2 \varphi}{\partial p^2} = 0. \quad (12.1)$$

Now suppose that at the lower boundary, the temperature consists of a (negative, shall we say) north-south temperature gradient plus superimposed perturbations:

$$\theta = \bar{\theta}_y y + \theta', \quad (12.2)$$

where $\bar{\theta}_y$ is the mean temperature gradient and that the wind consists of a zonal flow plus perturbations to it:

$$\mathbf{V} = u_0 \hat{i} + \mathbf{V}'. \quad (12.3)$$

The dynamical boundary condition, (11.1), becomes, after assuming that the perturbations are small and neglecting quadratic terms in the perturbation variables,

$$\left(\frac{\partial}{\partial t} + u_0 \frac{\partial}{\partial x}\right) \frac{\partial \varphi}{\partial p} + \gamma \frac{\partial \varphi}{\partial x} = 0 \quad \text{at} \quad p = p_0, \quad (12.4)$$

where we have used the geostrophic wind relation (10.6), and

$$\gamma \equiv -\bar{\theta}_y R / f_0 p_0, \quad (12.5)$$

using (8.33). (Had we used an example from the ocean, γ would have been $-G\bar{\sigma}_y$ evaluated at a suitable pressure level.)

For simplicity, we shall assume that the temperature perturbations vanish at the top of the fluid, so that

$$\frac{\partial \varphi}{\partial p} = 0 \quad \text{at} \quad p = 0. \quad (12.6)$$

Since the fluid is semi-infinite, we are entitled to look for model solutions that are periodic in x , y , and time, so let

$$\varphi = \hat{\varphi}(p) e^{ik(x-ct)+ily}, \quad (12.7)$$

where k , l , and c have the same meanings as before. Substituting (12.7) into (12.1), (12.4) and (12.6) gives

$$\frac{d^2 \hat{\varphi}}{dp^2} - \frac{\mathcal{S}}{f_0^2} (k^2 + l^2) \hat{\varphi} = 0, \quad (12.8)$$

subject to the boundary conditions

$$(u_0 - c) \frac{d\hat{\varphi}}{dp} + \gamma \hat{\varphi} = 0 \quad \text{on} \quad p = p_0, \quad (12.9)$$

$$\frac{d\hat{\varphi}}{dp} = 0 \quad \text{on} \quad p = p_t. \quad (12.10)$$

The system comprised of (12.8)–(12.10) is a closed eigenvalue problem with the solution

$$\hat{\varphi} = A \cosh(rp), \quad (12.11)$$

where A is an arbitrary amplitude, with

$$r^2 \equiv \frac{\mathcal{S}}{f_0} (k^2 + l^2). \quad (12.12)$$

This can only satisfy the lower boundary condition (12.9) if

$$c = u_0 + \frac{\gamma}{r} \operatorname{ctnh}(rp_0), \quad (12.13)$$

with r given by (12.12). Provided γ is positive (which, from (12.5), implies a negative temperature gradient), the waves will travel eastward relative to the background flow. Note that, like Rossby waves, the relative phase speed increases without bound as the horizontal wavelength becomes large. It is also clear from (12.11) that the wave amplitude decreases more or less exponentially upward (i.e., with decreasing pressure). The wave is three-dimensional, but trapped at the lower boundary. Note that all the time dependence is in the lower boundary condition, (12.9).

The wave dynamics are conceptualized in Figure 12.1b. Where the isotherms are deflected northward, the temperature perturbation is positive, and vice versa; invertibility sees those temperature perturbations as potential vorticity perturbations of the same sign, so that positive vorticity is associated with positive θ perturbations. The associated geostrophic flow, illustrated in Figure 12.1b, is in quadrature with θ in such a way that the wave propagates eastward relative to any background flow. Remember that there are no q_p perturbations in the interior of the fluid in the example, and that the boundary temperature anomalies act like delta function q_p anomalies, so the wave is evanescent with height away from the boundary. That is why the wave is called an *edge wave*.

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