

## Homework assignment 2

### Problem 1

Let  $v$  be a Gaussian variable. First show that all odd moments of  $v$  are zero. Then show that all even moments are related to the variance  $\sigma^2 = \langle (v - \langle v \rangle)^2 \rangle$  and write what is the relationship. In particular write the skewness and the kurtosis of a Gaussian variable.

### Problem 2

The logistic map presents the same closure problem encountered in turbulence. The equation for any moment of  $x$  depends on higher order moments and cannot be solved without additional information on these higher order moments. Consider for example the equation for the ensemble average of  $x$ ,

$$\bar{x}_{n+1} = r \bar{x}_n - r \overline{x_n^2}, \quad (1)$$

where the overbar denotes the ensemble average. The equation for the first moment,  $\bar{x}$ , depends on the second moment  $\overline{x^2}$ . Derive the equation for the second moment and shows that it depends on moments higher than second.

Most methods of closing the hierarchy of moments make assumptions about the relationship of  $(n+1)$ -th order moments to  $n$ -th order moments. A typical approach is to assume that the statistics of  $x$  are Gaussian and use the results of problem 1 to relate all moments to the second. Apply this method to close the equation for the second moment. With this assumption, the equation for the first and second moment form a closed set. Use MATLAB to iterate the two equations for  $r = 4$ . What happens? Can you explain why this approach fails [Hint: you might want to go back to Lecture 2 and remind yourself of what is the stationary distribution for the logistic map]?