Two-D Turbulence – Homework 2

Structure functions/ Covariances

For homogeneous, isotropic turbulence, we can write the covariance in terms of $f(r)$ which is the longitudinal covariance (the expected value of the production of the components of velocity along the the line joining the two points; i.e., parallel to $\mathbf{r}$) and $g(r)$ which is the transverse covariance (the expected value of the product of the velocities normal to $\mathbf{r}$). If $\mathbf{r} = r\hat{\mathbf{x}}$ then

$$\langle u(x,y)u(x+r,y)\rangle = f(r) \quad , \quad \langle u(x,y)v(x+r,y)\rangle = 0 \quad , \quad \langle v(x,y)v(x+r,y)\rangle = g(r)$$

Why is the second true?

a) Use these to show in general that

$$\langle u_i(\mathbf{x})u_j(\mathbf{x}+\mathbf{r})\rangle = \frac{f(r)-g(r)}{r^2}r_ir_j + g(r)\delta_{ij} \tag{1}$$

b) Applying the continuity equation to the covariance implies

$$\frac{\partial}{\partial r_j}\langle u_i(\mathbf{x})u_j(\mathbf{x}+\mathbf{r})\rangle = 0$$

Use this and eqn. (1) to find the relationship

$$\frac{\partial}{\partial r}(rf) = g$$

in two dimensions. (What is it in three?)

c) In two dimensions, the flow is given by a streamfunction

$$u = -\frac{\partial}{\partial y}\psi \quad , \quad v = \frac{\partial}{\partial x}\psi$$

so that we can relate the transverse and longitudinal covariances to $C(r) \equiv \langle\psi(\mathbf{x})\psi(\mathbf{x}+\mathbf{r})$. Use again $\mathbf{r} = r\hat{\mathbf{x}}$ to find the relationship between g and C.

d) From the `t256` turbulence simulation, find C and estimate f and g from that. The MATLAB program `qgproc2` gives a start on the problem. I've looked at about $t = 8$, but you can look at other times as well. The `qgprocm.m` program will display the fields. The files are available on-line as

`http://lake.mit.edu/~glenn/12.822t/t256.in`
`http://lake.mit.edu/~glenn/12.822t/t256.out`
`http://lake.mit.edu/~glenn/12.822t/qgprocm.m`
`http://lake.mit.edu/~glenn/12.822t/qgproc2.m`

e) What does $S_2(r)$ look like and what do you think about the practicalities of computing of $S_p(r)$?

f) In your copious free time, you can look at the runs with beta (`t256b1` and `t256b5`) to see what differences there might be and how anisotropy shows up. [optional...]