12.864 Inference from Data and Models 25 April 2005
Problem Set No. 5 Due: 11 May 2005

1. Let $\Delta t = 0.1$. Generate, numerically, a sine wave $x_n = 2.5 \sin(2\pi n \Delta t / 64)$, $0 \leq n \leq 1023$.

(a) Using a numerical code of your choosing, compute the Fourier transform, and show that you can identify properly the frequency, amplitude, and phase of x_n.

(b) Do the same for $y_n = 5 \cos(2\pi n \Delta t / 64)$, $0 \leq n \leq 1023$.

(c) From the numerical results in (a), (b) find the Fourier *series* coefficients of x_n, y_n in both complex and real form.

(Use a properly labelled plot to show me the results, not long lists of numbers, please.)

2. Let θ_n be a set of pseudorandom Gaussian variates with unit variance and zero mean at the same times as in Problem 1. Let a_p, b_p be the real and imaginary parts of its Fourier series. Compare, in a plot, $a_p^2 + b_p^2$, with $|\hat{\theta}_p|^2$, the magnitude squared of the Fourier transform.

3. Re-do Problem 2 but for $y_n + \theta_n$ and interpret the result.

4. Now let $x_n = 0.9 x_{n-1} + \theta_n$ where $x_0 = 0$, over the same time interval. (a) Compute and display $|\hat{x}_p|^2$. Compute and display a local frequency band average (e.g., over 8 neighboring frequencies) $|\hat{x}_p|^2$ and describe the result. (b) Do the same for $x_n + y_n$.

Optional but useful: 5. Estimate the coherence between x_n in the last problem and θ_n.