14.02 Quiz 3 Solutions
Fall 2004

Multiple-Choice Questions

1) According to the article, what is the main reason investors in US government bonds grow less optimistic?

   A) They are concerned about the decline (depreciation) of the dollar, which, in the long run, leads to an increase in the price level.
   B) They are expecting high inflation due to rising oil prices, which would lead to a fall in real interest rates in the future, despite the Fed's tightening.
   C) Because the Fed is in "tightening mode," due to reports of higher job creation and evidence of low inflation, and thus they expect the price of bonds to decrease.
   D) Because the Fed is in "tightening mode," due to reports of higher job creation and evidence of low inflation, and thus they expect the price of bonds to increase.
   E) Both A) and D).

   Answer: ***C)****. The investors are growing more bearish because the Fed is likely to keep raising interest rates despite the decline of the dollar. The fact that job creation has picked up and that inflation is not a problem at the moment points to the strengthening of the economy. So, there is no reason for the Fed to stop tightening. What happens if interest rates rise? We know from the textbook chapter on valuation of securities that bond prices are inversely related to interest rates. As interest rates increase, treasuries lose their value (their prices decrease). This might seem good from a perspective of a buyer of a bond. But remember that the treasuries investors in the articles are already holding the securities! Obviously, they are not happy about the decrease in the value of their holdings.*

2) The report says that "A Bloomberg News survey last week showed that most experts expected the Fed to raise the target level for overnight loans between banks to 2.25 percent from 2 percent on Dec. 14. A month ago, only a few forecast an increase in the federal funds rate." This information implies the valuation of any assets (hint: use the formula for EPDV discussed in class and assume nothing else changes in reaction to what the Fed does):

   A) Will decrease on Dec. 14 if the Fed increases the target level to 2.25 percent.
   B) Will decrease on Dec. 14 if the Fed raises the target level to 2.50 percent.
   C) Have decreased from last month to today.
   D) Both A) and C).
   E) Both B) and C).

   Answer: ***E)****. V is a negative function of $i^e(t+s)$. The report suggests that i^e has increased during the last month and is expected to increase on Dec. 14, therefore V has decreased during the last month (thus, option C) is correct) , if the Fed increases the interest rate as expected nothing else will change (therefore A) is not correct).*

And if the Fed increases the interest rate by more than expected, assets prices will decrease again (thus B) is also correct). Then, the correct answer is E).

3) The article reports that the dollar weakened 3.9% against the euro over the last month. If the dollar continues to weaken, we should expect (assume that the Marshall-Lerner condition holds)

A) Exports from the US to Europe to increase.
B) US imports from Europe to increase.
C) The US trade balance with Europe to deteriorate.
D) Both A) and C).
E) Both B) and C).

Answer: ***A)****. The depreciating dollar will weaken the purchasing power of US consumers vis-à-vis European goods, which will lead to a decline in imports. The US trade balance will improve under Marshall-Lerner conditions as a result of higher exports and lower imports.*

4) The article mentions the"China Scare," stating that "US government bonds fell on Nov. 26 after China Business News reported Yu Yongding, a Chinese central bank official, said China had cut its holdings of U.S. debt. Why did the price of US government bonds fall as a reaction to the news from China?

A) The demand for dollars declined which has to lead to an increase in current US interest rates.
B) The demand for dollars declined which has to lead to an increase in current and future US interest rates.
C) US interest rates are expected to rise to prevent the US from running into problems with financing its trade deficit.
D) The Fed will have to raise interest rates to prevent the US government from selling its bonds.
E) Bankers in New York were worried that the report was actually distorted which leads to uncertainty about the true demand for dollars.

Answer: ***C)****. When a country has a trade deficit, it needs to borrow funds from abroad to finance this deficit. The fact that China holds U.S. government bonds helps to finance the U.S. trade deficit. For example, the U.S. buys more goods from China than China buys from the U.S. and thus transfers more dollars to China than the Chinese need to buy U.S. goods. China accepts these additional dollars since they use them to buy U.S. government bonds (that is the U.S. borrows this money from China since bonds are just IOUs). If China suddenly decides that it wants to buy fewer U.S. government bonds, the U.S. won't be able to borrow as much from China, unless the U.S. increases the interest rate on government bonds. Increasing the interest rate would make U.S. government bonds more attractive as an investment since they would pay a higher return.*

Long Question I (35/100 points)
Open Economy AS-AD and Growth

Assume that the economy is described by the following set of equations.

Exchange rate: $E = \overline{E}$

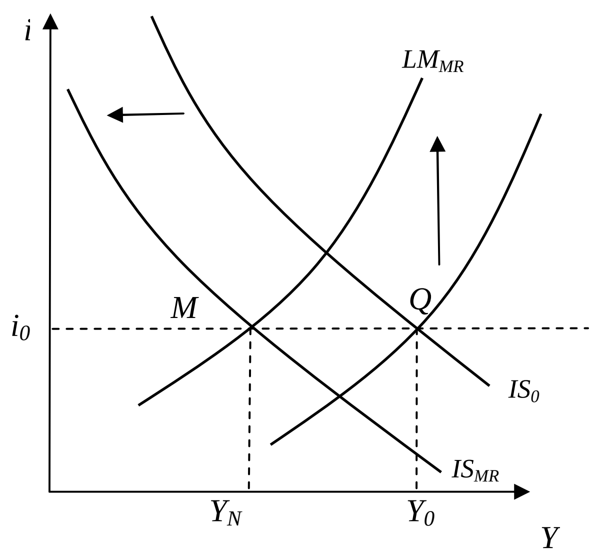
Price Setting: $P = (1+\mu)\left(\frac{W}{A}\right)$

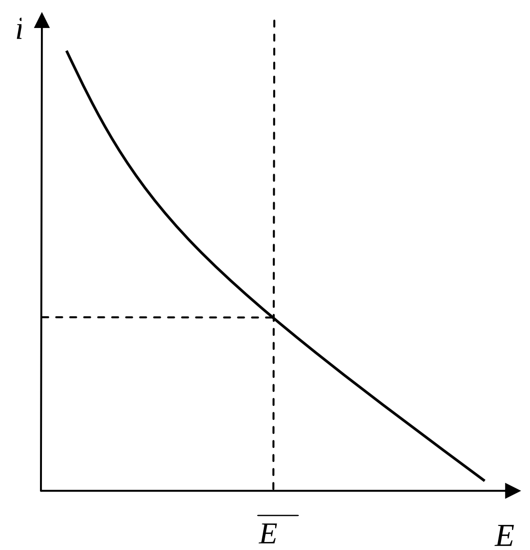
Wage Setting: $W = P^e A^e F(u,z)$

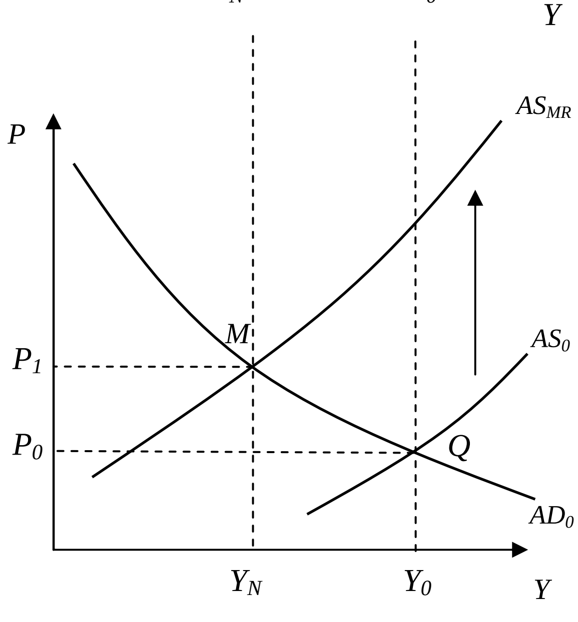
AD: $Y = C(Y, T) + I(Y,i) + G + NX(Y, Y^*, \varepsilon)$

Assume that the Marshall-Learner Condition is satisfied.

1. Suppose the economy is at a place where $u < u_N$ (point Q). Assume that A and A^e are constant. Without fiscal policy and monetary policy interventions, what happens over time? Show graphically. Label all curves. Label the medium-run equilibrium as point M clearly.

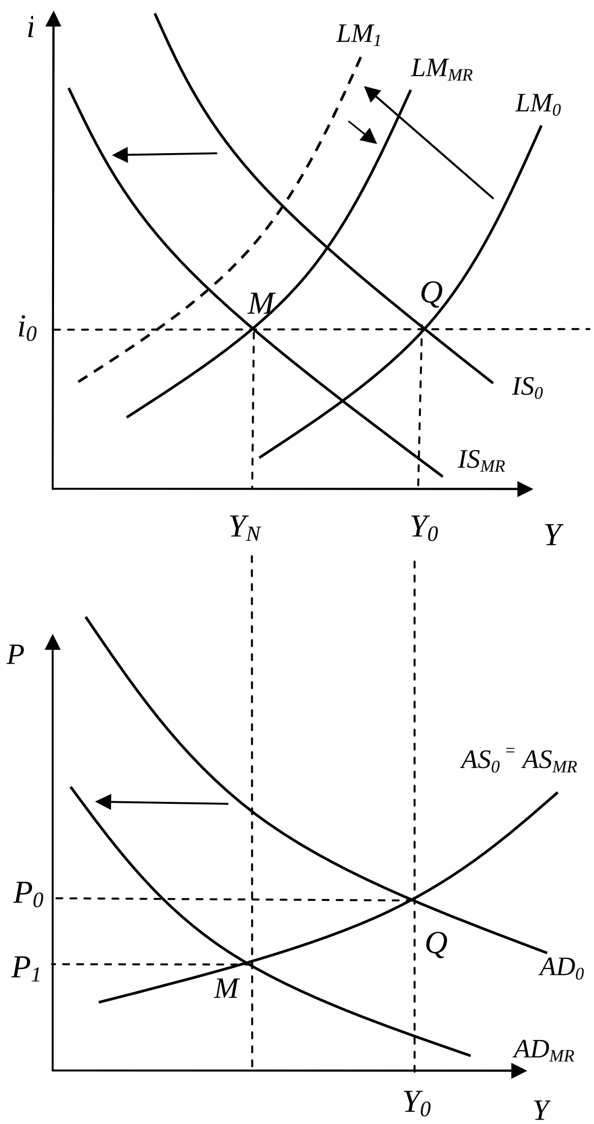


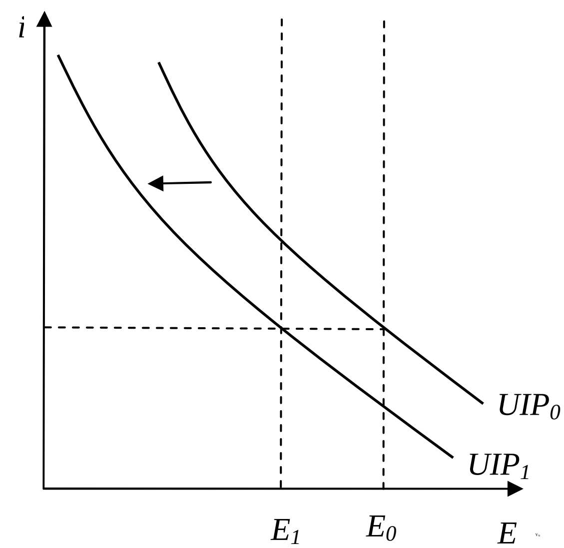




At point Q, $Y_0 > Y_N$, so $u_0 < u_N$. This means that the economy is at a level that is beyond its capacity. So, people expect the price level to increase in the near future. When P^e increases, W increases since the wage setting relation tells us that $W = P^eA^eF(u,z)$. An increase in W increases P since the price setting relation tells us that $P = (1+\mu)(W/A)$. As P increases, the AS curve shifts to the left and up. At the same time, LM shifts up, because an increase in P decreases the real money *supply even though the Fed is not decreasing nominal money supply. This process continues until the economy reaches point M which is the medium-/long-run equilibrium where $Y=Y_N$ and $u=u_N$.*

2. If the central bank announces a one-time revaluation of its currency that is credible, what happens over time? Still, assume that A and A^e are constant. Show graphically. Label all curves. Label the medium-run equilibrium as point M clearly.





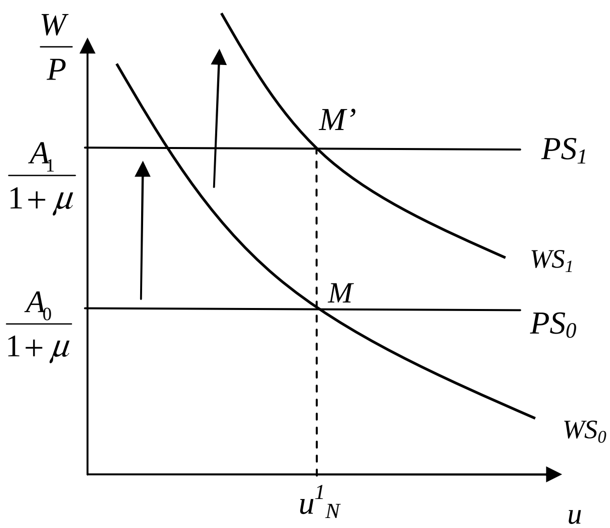
If a country has a credible fixed exchange rate system, then its announcement of a one-time revaluation is also credible. So, if the central bank announces a one-time revaluation of its currency, it is believed by the investors. Credibility means that investors expect the exchange rate of this country to decrease and be fixed at its new level. So, E^e decreases by the amount of the announcement. This shifts the interest-rate parity condition curve to the left/down. Since the central bank has not changed the nominal money supply, today's exchange rate increases to $\bar{E}_1$ and $i=i^*$.

When the exchange rate decreases, net exports (NX) decrease due to the Marshall-Lerner condition. This means that the IS curve will shift to the left. This also shifts the AD curve to the left, and P starts to fall. In a fixed exchange rate regime, monetary policy must accommodate. The central bank now must decrease the money supply so that the exchange rate does not deviate from $\bar{E}_1$. Notice that the central bank actually decreases the money supply, so that the LM curve shifts to LM_1, but the *because of a decrease in P, the LM curve is LM_{MR} in the medium-/long-run. Point M is the medium-/long-run equilibrium.*

3. What is the advantage of government intervention, namely one-time revaluation of the domestic currency in question 2, if the speed of adjustment from point Q to the medium-run equilibrium was the same as in question 1? Limit your answers to a few sentences.

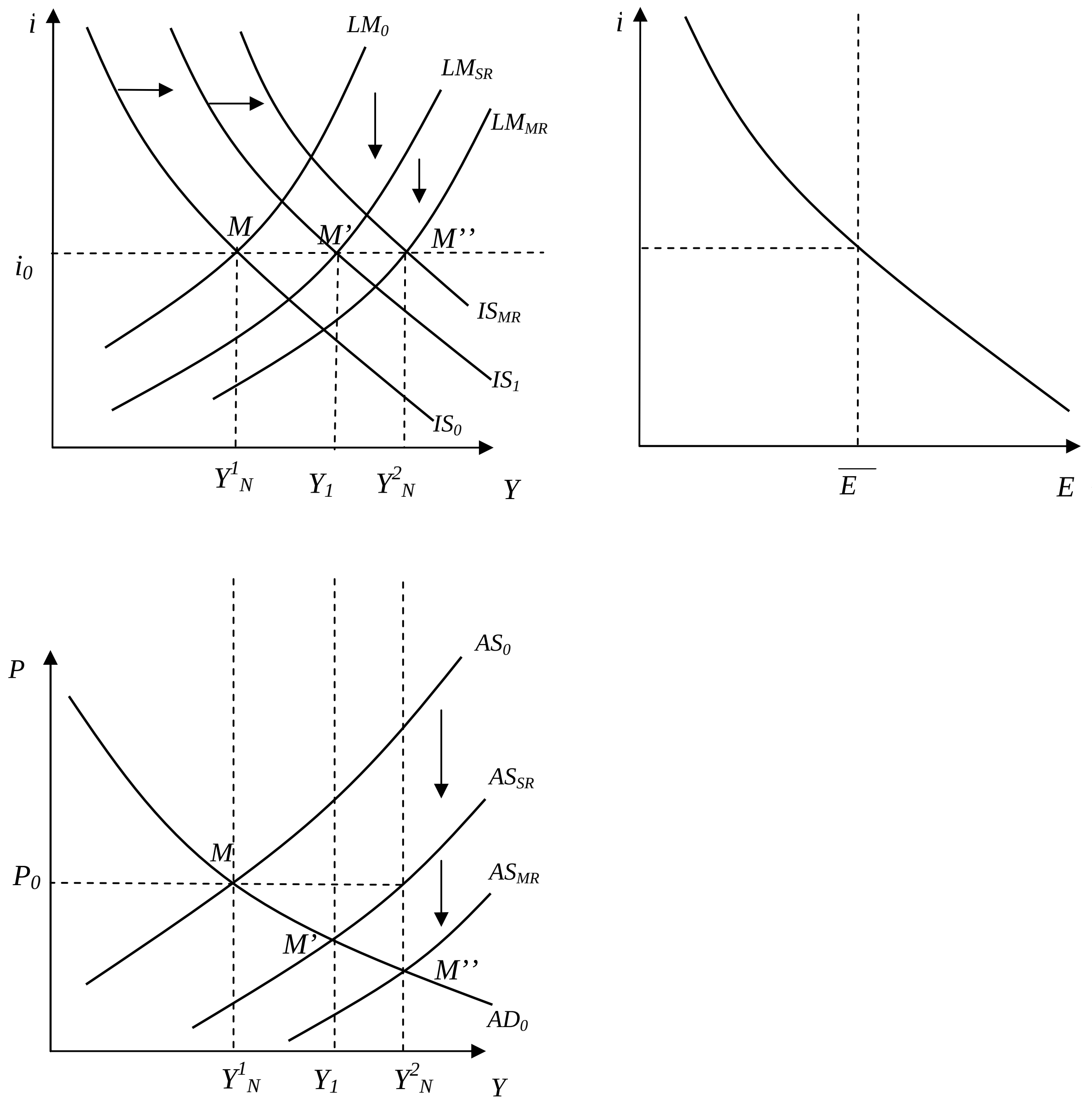
   The main difference between the medium-run equilibrium of question 1 and 2 is the equilibrium price. Even if the speed of adjustment is the same (usually the adjustment in question 2 is faster), the medium-run equilibrium price is higher if the government (and the central bank) does not intervene. This means that inflation is higher under the scenario in question 1. This problem of inflation does not exist with one-time revaluation.

4. Now, the economy is at the medium/long-run equilibrium (point M) but it experiences an increase in productivity. What happens to its natural rate of unemployment if people's expectations about prices and productivity are correct? Why? Show graphically. Label all curves.



When this economy experiences an increase in productivity, nothing happens to its natural rate of unemployment. This is because both the PS schedule and the WS schedule shift up by the same amount. Why by the same amount? It is because people's expectations of productivity are equal to the actual productivity improvement. The WS curve shifts up because an increase in A^e increases W and this increases the real wage (W/P). The PS curve shifts up because an increase in A means that P decreases since less labor is required for production. The decrease in P increases the real wage.

5. As in question 4, the economy is at the medium-run equilibrium (point M), but it experiences an increase in productivity which does not affect AD. If people's expectations about productivity are always correct, what happens in the short- and medium-run? Show graphically. Label all curves. Label short- and medium-run equilibria clearly. (11 points) (hint: u = 1 – Y/AL)



In the short-run:
When productivity increases (an increase in A), the AS curve shifts down and to the right. This lowers P which has 2 effects. First, the real money supply increases even though the nominal money supply has not changed. This shifts the LM curve down. At the same time, a decrease in P leads to a real depreciation of the domestic currency. This increases net exports due to the Marshall-Lerner condition. So, the IS curve shifts to the right also. The intersection of IS-LM must be at point M'. Why? A fixed exchange regime means that $i = i^*$ *and* $E = \bar{E}$*. (Note: If the decrease in P is not enough (or too much) to increase real money supply which would correspond to* LM_{SR}*, then the central bank must either increase (or decrease) nominal money supply so that* $i=i^*$ *is maintained, since that's what it means to be in a fixed exchange regime.)*

Another important point here is that with a higher A (productivity), does not change the natural rate of unemployment since people's expecations were correct. However, $u=1-(Y/AL)$*. So, even though* u_N *is constant,* Y_N *increases.*

Short-run to medium-run:
At point M', the economy is performing at a level below that of the natural level of output. So, people's expectations of prices start to decrease. As P^e *decreases, the AS curve starts to shift to the right/down until it reaches point M''. As the AS curve moves toward point M'', the actual price level also drops. This increases real money supply (the LM curve shifts down) and the real exchange rate (IS shifts right). Again, since* $i=i^*$ *at all times, we know that at point M'', the IS and the LM curves must intersect, and the AS and the AD curves must also intersect.*

Long Question II (45/100 points)
Growth

The Republic of Solowakia has the following production function:
$Y_t = F(K_t, N_t) = A_t K_t^{\alpha} N_t^{1-\alpha}$, where $\alpha<1$.
Assume for now that A_t is constant over time (there is no technological progress in this economy, so $A_t=A$), g_N is the growth rate of N, δ is the rate of depreciation in this economy, and s is the saving rate.

1. Verify that the above production function has the property of constant returns to scale and rewrite the production function in terms of only capital per worker. (Define $k_t = \frac{K_t}{N_t}$ and $y_t = \frac{Y_t}{N_t}$.)

   First, we must verify that $f(\lambda K_t, \lambda N_t) = \lambda f(K_t, N_t)$, that is, if you multiply all inputs by a scalar, you will end up multiplying output by the same amount.

   $f(\lambda K_t, \lambda N_t) = A(\lambda K_t)^{\alpha}(\lambda N_t)^{1-\alpha} = \lambda^{\alpha+1-\alpha} AK_t^{\alpha}N_t^{1-\alpha} = \lambda AK_t^{\alpha}N_t^{1-\alpha} = \lambda f(K_t, N_t)$.

   To write the production function in intensive form, let $\lambda=1/N$.

   $$F(K_t,1) = A\left(\frac{K_t}{N_t}\right)^{\alpha}\left(\frac{N_t}{N_t}\right)^{1-\alpha} = A\left(\frac{K_t}{N_t}\right)^{\alpha}$$

   Define f so that $f(k_t) \equiv F(k_t,1)$.
   Then, $y_t = f(k_t) = A\, k_t^{\alpha}$. *(Recall that A is just a constant in this model!)*

2. Solve for the steady state values of capital per worker (k*), output per worker (y*), and consumption per worker (c*). Draw a diagram that shows all three steady state values you calculated.

   Recall that the steady state is given by the crossing of the investment and the required investment schedules. That is, $sf(k^*)=(g_N+\delta)k^*$. (See page 225.)

   $$\Rightarrow k^* = f(k^*)\left(\frac{s}{g_N+\delta}\right)$$

   $$k^* = A(k^*)^{\alpha}\left(\frac{s}{g_N+\delta}\right)$$

   $$(k^*)^{1-\alpha} = A\left(\frac{s}{g_N+\delta}\right)$$

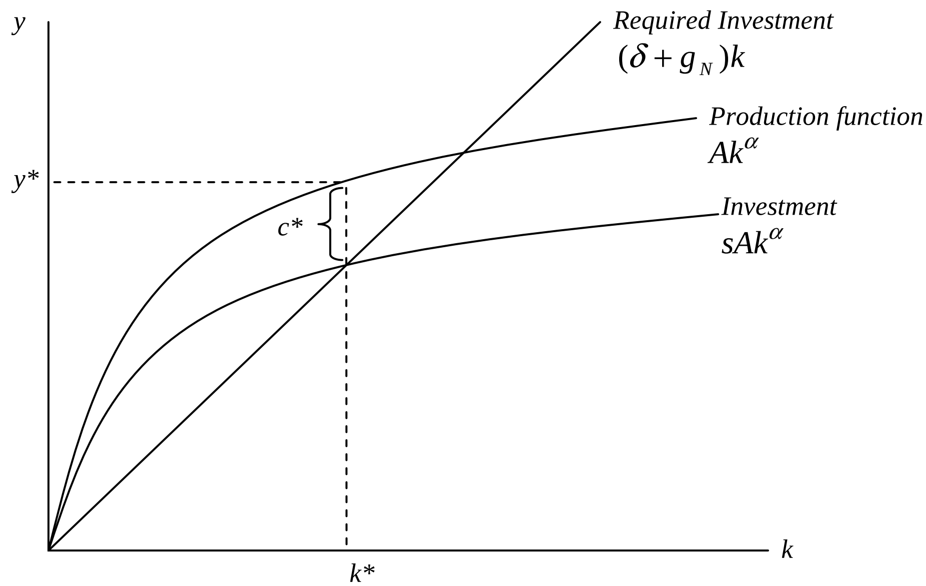
   $$k^* = A^{\frac{1}{1-\alpha}}\left(\frac{s}{g_N+\delta}\right)^{\frac{1}{1-\alpha}}$$

   Plugging this k into the production function, we get y*:*

$$y^* = A(k^*)^{\alpha} = A(A^{\frac{1}{1-\alpha}}(\frac{s}{g_N+\delta})^{\frac{1}{1-\alpha}})^{\alpha} = A^{\frac{1}{1-\alpha}}(\frac{s}{g_N+\delta})^{\frac{\alpha}{1-\alpha}}$$

$c_t = y_t(1-s)$

$$c^* = A^{\frac{1}{1-\alpha}}(\frac{s}{g_N+\delta})^{\frac{\alpha}{1-\alpha}}(1-s)$$



3. Find the saving rate at which steady-state consumption is maximized (i.e. we are at the Golden Rule steady state).

There are a few ways of doing this, but here we will maximize steady-state consumption by setting $\frac{\partial c^*}{\partial k^*}$ equal to zero.

Note that $c^* = y^* - sy^* = y^* - (\delta + g_N)k^*$

$$\frac{\partial c^*}{\partial k^*} = \frac{\partial y^*}{\partial k^*} - (\delta + g_N)$$

$$\frac{\partial y^*}{\partial k^*} = (\delta + g_N)$$

$$\frac{\partial y^*}{\partial k^*} = A\alpha(k^*)^{\alpha-1}$$

$$A\alpha(A^{\frac{1}{1-\alpha}}(\frac{s}{g_N+\delta})^{\frac{1}{1-\alpha}})^{\alpha-1} = \delta + g_N$$

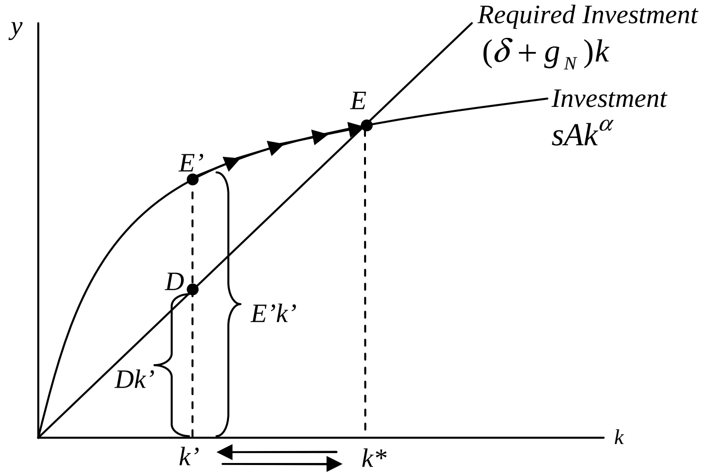
$$\alpha(\frac{g_N+\delta}{s}) = \delta + g_N$$

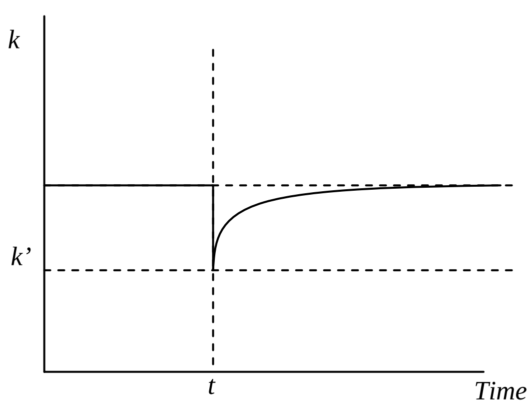
$s = \alpha$.

This says that the optimal level of savings is equal to the share of capital in the production function, α. *The intuition is that diminishing returns reduce the usefulness of additional units of capital, so investing more is not always optimal.*

4. Suppose that at time t there is a one-time inflow of foreign workers into the country, so that N jumps from N_0 to N_1. (Assume that this does not affect g_N.) Draw two diagrams: one showing what happens to the investment and required investment schedules, including dynamics, and one depicting the effects of this inflow on capital per worker over time.

   An inflow of foreign workers is equivalent to an increase in N. Therefore, K/N decreases (k decreases). So, in this case, capital per worker would (immediately) jump down to a level such as k' in the short-run. However, in the long run, the dynamics will bring the economy back to the original steady-state level of capital per effective worker, k_1. Why? When there is an increase in N, we end up at point E', where investment per worker equals the vertical distance E'k'. The amount of investment required to maintain that level of capital per worker is clearly smaller than the amount E'k' (distance Dk'). Because actual investment exceeds investment that is required to maintain the existing level of capital per worker at E', k increases. Hence, starting from k', the economy moves to the right, with the level of capital per worker increasing over time. This continues until investment per worker is just sufficient to maintain the existing level of capital per worker, that is until we return to the initial steady-state, E. (See page 248.) So, the effect of immigration will only be temporary (because nothing happened to investment or required investment).*





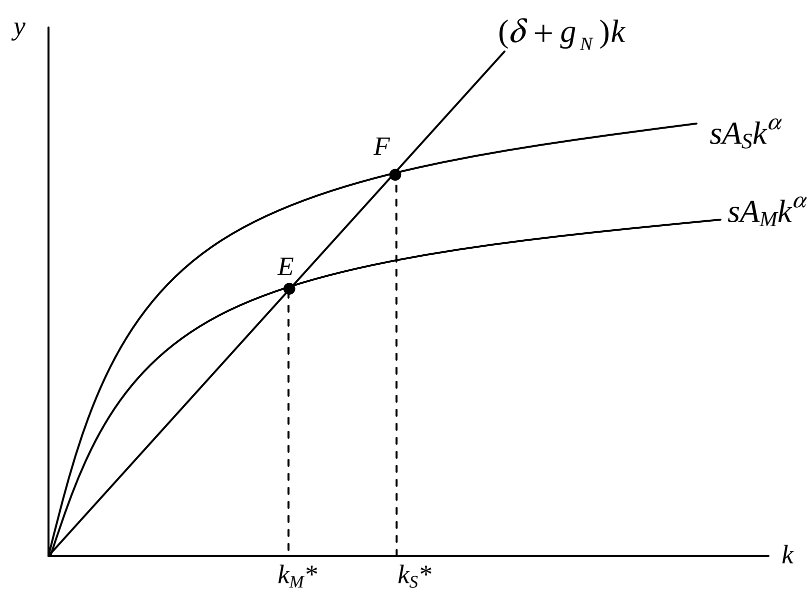
The figure on the left depicts the evolution of capital per worker over time. Prior to time t, capital per worker is at the level k. At time t, when there is an inflow of workers into Solowakia, capital per worker immediately drops to k'. Then, over time, capital per worker increases back to the original steady-state level, k*.*

5. Suppose Solowakia (S) and Macronesia (M) have identical production functions and same δ, g_N, and s. However, A_S>A_M. Which country will have a higher steady-state capital per worker? Prove your answer mathematically and with a diagram.

$k^* = A_i^{\frac{1}{1-\alpha}} \left(\frac{s}{g_N+\delta}\right)^{\frac{1}{1-\alpha}}$, *where i={S,M}*

$\frac{\partial k^*}{\partial A_i} = \frac{1}{1-\alpha} A_i^{\frac{\alpha}{1-\alpha}} \left(\frac{s}{g_N+\delta}\right)^{\frac{1}{1-\alpha}} > 0$

(Because α<1 and all the other parameters are positive)
Therefore, as A increases, k increases. So, the country with a higher technological parameter, A, will have a higher steady-state level of capital per worker (which, in this case, is Solowakia). Intuitively, this follows from the fact that this technological change increases the marginal product of capital at every level of per worker capital stock.*



6. Assume that all countries are heading towards the same steady state (that is, in the long run, all countries have access to the same technology and have the same preferences as manifested in the same saving rate and population growth rate). Does the model predict growth for poorer countries should be faster, slower, or the same as richer countries? Show mathematically. (Hint: Define the growth rate of capital as $g_K = \frac{\Delta k_t}{k_t}$.)

   In this model, a poor country is poor because its capital per worker (k) is further below the steady-state value than is the capital per worker of a rich country (i.e. the marginal product of capital is greater in the poor country). This also means that its income per worker (y) is further below the steady-state income per worker.

   Countries that are approaching the steady state from below (which is true for poor countries) grow according to the excess of actual investment over investment that is required to maintain existing level of capital. The greater the difference, the faster is growth. That is, countries that start out with lower k ("poor countries") grow faster than countries with k closer to the steady state k ("richer countries").

   Mathematically:

   $$\frac{K_{t+1}}{N_t} - \frac{K_t}{N_t} = k_{t+1} - k_t = \Delta k_t = sy_t - (\delta + g_N)k_t$$ *(See page 223, equation 11.2.)*

   Dividing both sides by k_t:

   $$\frac{\Delta k_t}{k_t} = g_K = s\frac{y_t}{k_t} - (\delta + g_N)$$

   Recall that in part 1 we found that $y = f(k) = A\, k^{\alpha}$.

   $$g_K = sA\frac{k_t^{\alpha}}{k_t} - (\delta + g_N) = sAk_t^{\alpha-1} - (\delta + g_N)$$

   We see that since $0<\alpha<1$, g_K is decreasing in k (i.e. $\frac{\partial g_K}{\partial k_t} = sA(\alpha - 1)k_t^{\alpha-2} < 0$).

7. Suppose that α=0.5 in the given production function. Assume that the level of technology in the country depends on capital per worker, in particular $A=k^{\beta}$. Discuss convergence and growth in an economy with β=0.5 and compare it to an economy with β<0.5. Use diagrams and words.

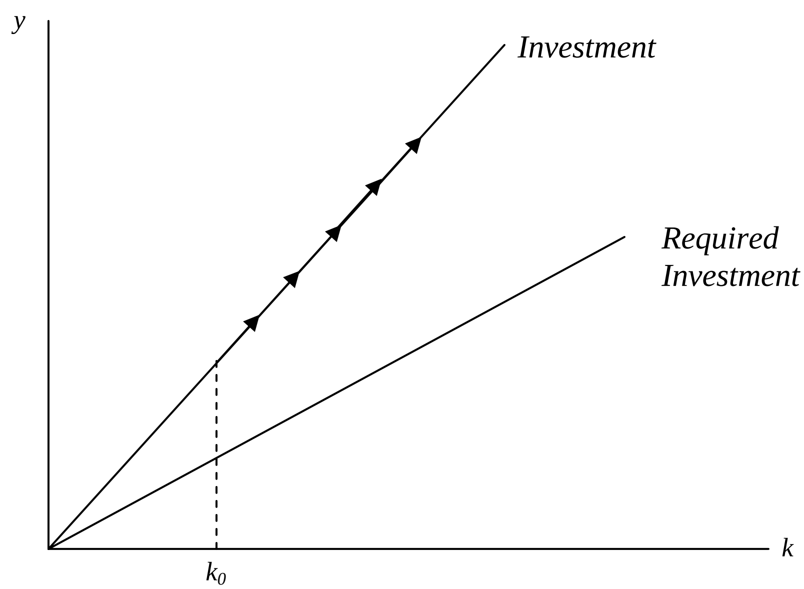
   For this economy, $y_t = k_t^{\beta + 0.5}$
   Convergence therefore will depend on the value of β. If the investment function, that is $sk^{\beta + 0.5}$, is concave in capital per worker, then there exists a steady state towards which economies with similar saving rates, technology growth rates, and depreciation rates will eventually converge. If it is convex, we will not converge to the steady state (the steady state will be unstable).
   For what values of β will the function be concave/convex? We can look at the second partial with respect to capital per worker, as we did on the problem set:

$$\frac{\partial y_t}{\partial k_t} = (\beta + 0.5) k_t^{\beta - 0.5}$$

$$\frac{\partial^2 y_t}{\partial k_t^2} = (\beta + 0.5)(\beta - 0.5) k_t^{\beta - 1.5}$$

This last expression is negative (and thus the function is concave) if and only if β<0.5. For β>0.5, it is convex. When β=0.5, the function is a line.

In particular, the following diagram describes the economy when β=0.5.



If the country starts out at the level of capital per worker such as k0, it will grow forever. This is true for any starting level of capital per worker. In this sense, the economy will never converge to a steady state. The reason is that the production function is linear in capital, and therefore it does not exhibit diminishing returns to capital.

What about growth in this economy? We can use the same logic as in part 7 to answer this question.

$$k_{t+1} - k_t = \Delta k_t = s y_t - (\delta + g_N) k_t$$

$$\Delta k_t = sAk_t - (\delta + g_N) k_t$$

Dividing both sides by k_t:

$$\frac{\Delta k_t}{k_t} = g_K = sA - \delta - g_N$$ *, which is a constant.*

This shows that k grows at a constant rate. So, this is a model of endogenous growth, because it generates steady growth even without technological progress. In contrast to the Solow model, growth depends, even in the long run, on the saving rate.

For β<0.5, there exists a steady state to which economies will eventually converge. To find it we again set $sf(k^*)=(g_N+\delta)k^*$.

$$k^* = \frac{s(k^*)^{\beta + 0.5}}{g_N + \delta}$$

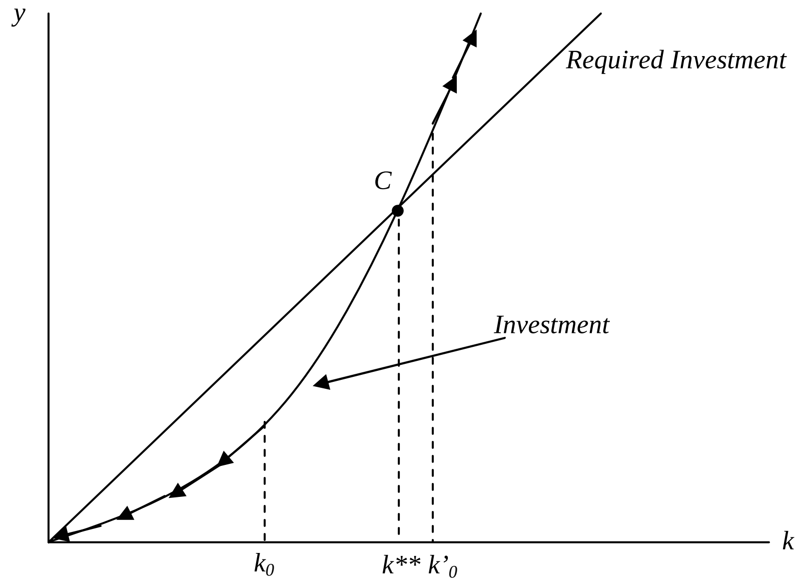
$$(k^*)^{0.5-\beta} = \frac{s}{g_N + \delta}$$

$$k^* = \left(\frac{s}{g_N + \delta}\right)^{\frac{1}{0.5-\beta}}$$

$$y^* = (k^*)^{\beta+0.5} = \left(\frac{s}{g_N + \delta}\right)^{\frac{0.5+\beta}{0.5-\beta}}$$

We have the standard diagram. (See solution to part 4.)
In this model, just as in the standard Solow model, there is convergence to the steady-state.

The rest of the solution is just F.Y.I.
What about when β>0.5? Then the diagram becomes:



*The steady state C is locally unstable. That is, if we start at any level of capital per worker below k**, the economy will shrink to nothing (illustrated by the move from k_0 to zero). So, this is like a poverty trap – the country that starts out very poor not only stays poor, but grows poorer over time. Why? Because at k_0, the amount of investment required to maintain that level of capital per worker exceeds actual investment. Therefore, k decreases, and the economy moves to the left, with the level of capital per worker decreasing over time.*
*However, if the economy starts at a level of capital per worker above k**, then the economy will grow forever. This is because at k'_0, actual investment exceeds the amount of investment required to maintain that level of capital per worker. Therefore, k increases, and the economy keeps moving to the right, with the level of capital per worker increasing over time.*
In this sense, there is no convergence for an economy with $\beta \geq 0.5$.