

## 14.02 Principles of Macroeconomics

Spring 2014

Problem Set 5 Solutions

Due: May 5, 2014

### 1 Consumption and Saving with Uncertain Income (20 pts)

Consider a consumer who lives for three periods: youth, middle age and old age. When young, the consumer earns \$20,000 in labor income. Earnings during the middle age are uncertain: there is a 50% chance that the consumer will earn \$40,000 and a 50% chance that the consumer earns \$100,000. When old, the consumer spends savings accumulated from the previous periods. Assume that inflation, expected inflation, and the real interest rate are equal to zero. Ignore taxes for this problem.

a. What is the expected value of lifetime earnings in the middle period of life? Given this number, what is the present discounted value of the expected lifetime labor earnings? If the consumer wishes to maintain constant expected consumption over her lifetime, how much will she consume in each period? How will she save in each period? (6 pts)

Answer:

Expected income at middle age is  $0.5 \times 40,000 + 0.5 \times 100,000 = 70,000$ . Since both real interest rate and inflation are zero, the discount factor is equal to one. Thus the present value of expected lifetime labor earnings is  $20,000 + 70,000 = 90,000$ . If he wants to maintain a constant expected consumption during lifetime, he should spend 30,000 in each period. This means that the consumer will save  $-10,000$  in the first period (negative sign means borrow), and 30,000 in expectation in the second period.

b. Now suppose that the consumer wishes to maintain a minimum consumption level of \$20,000 in each period of her life. To do so, she must consider the worst outcome. If earnings during the middle age turn out to be \$40,000, how much should the consumer spend when she is young to guarantee consumption of at least \$20,000 in each period? How does this level of consumption compare to the level you obtained for the young in part a?

(7 pts)

Answer:

In the worst case, the consumer receives 20,000 in the first period and 40,000 in the second period. This is just enough to maintain 20,000 consumption in each period. Therefore, the consumer should only consume 20,000 in the first period, which is lower than the value obtained in part a.

c. Given your answer in part b, suppose that the consumer earnings during middle age turn out to be \$100,000. How much will she spend in each period of her life? Will consumption be constant over the consumer lifetime? (Hint: When the consumer reaches middle age, she will try to maintain constant consumption for the last two periods of life, as long as she can consume at least \$20,000 in each period) (7pts)

Answer:

Given the consumer already spends 20,000 for consumption in the first period, it will split 100,000 between the last two periods. Therefore, it spends 50,000 in each of the last two periods.

## 2 Ricardian Equivalence and Fiscal Policy (30 pts)

First consider an economy in which Ricardian equivalence does not hold.

a. Suppose the government starts with a balanced budget. Then, there is an increase in government spending, but there is no change in taxes. Show in an IS-LM diagram the effect of this policy on output in the short run. How will the government finance the increase in government spending? (5 pts)

Answer:

The IS curve shifts to the right due to the increase in government spending. As a result, there is an increase in output in the short run. Because government doesn't change taxes, the increase in government spending is financed through debts (the government borrows from the private sector).

b. Suppose, as in part (a), that the government starts with a balanced budget and then increases government spending. This time, however, assume that taxes increase by the same amount as government spending. Show in an IS-LM diagram the effect of this policy on output in the short run. How does the output effect compare with the effect in part (a)? (5 pts)

Answer:

The IS curve shifts to the right, thus there is an increase in output in the short run. The increase in output is smaller compared with the increase in part (a), because tax is also increased. (Remember that a budget balanced policy is output boosting with a multiplier equaling to one.)

Now suppose Ricardian equivalence holds in this economy.

c. Consider again an increase in government spending with no change in taxes. How does the output effect compare to the output effects in parts (a) and (b)? (10 pts)

Answer:

When Ricardian equivalence holds, even taxes remain the same, consumers will anticipate higher taxes and thus lower disposable income in future. This anticipation effect makes consumers consume less and save more. Therefore, the boost in output is smaller than part (a).

The boost in output would be larger than part (b), the intuition here is subtle.

First, Ricardian equivalence implies that how government finances its spending doesn't matter. Therefore increasing government spending with no change in taxes (finance it by increasing debt) is equivalent to financing the increased government spending by increasing taxes of the same amount.

Therefore, we can think of part (c) as a version of part (b) in a Ricardian equivalence context. Individuals are forward looking in part (c) (because Ricardian equivalence holds), thereby for the same decrease in current disposable income, they would cut consumption less. This is because forward-looking individuals have incentive to smooth consumption over time. The optimal consumption plan following a decrease in current disposable income is to cut consumption not only in current period (by a smaller amount) but also in all future periods.

While in part (b), individuals do not have such "consumption smoothing" incentive, hence the cut in consumption is bigger.

d. Consider again an increase in government spending combined with an increase in taxes of the same amount. How does this output effect compare to the output effects in parts (a) and (b)? (10 pts)

Answer:

The output effect is the same to part (c) because Ricardian equivalence holds. Thus the output effect is smaller than part (a) but is larger than part (b).

### 3 Debt and deficit (20 pts)

Consider an economy characterized by the following facts:

- i. The debt-to-GDP ratio is 40%.
- ii. The primary deficit is 4% of GDP.
- iii. The normal growth rate is 3%. (This is the growth rate of GDP)
- iv. The real interest rate is 3%, and there is no inflation.

a. Using your favorite spreadsheet software, compute the debt-to-GDP ratio in 10 years, assuming that the primary deficit stays at 4% of GDP each year; the economy grows at the normal growth rate in each year; and the real interest rate is constant, at 3%. (5 pts)

Answer:

The formula you need to use is

$$B_t = (1 + r)B_{t-1} + \text{primary deficit}$$

Since we only care about the debt to GDP ratio, the exact level of GDP doesn't matter. For simplicity, assume that the GDP in year 0 is 1.

|    | GDP     | primary deficit | debt    | debt/GDP ratio |
|----|---------|-----------------|---------|----------------|
| 0  | 1       | 0.04000         | 0.40000 | 0.4            |
| 1  | 1.03    | 0.04120         | 0.45320 | 0.44           |
| 2  | 1.0609  | 0.04244         | 0.50923 | 0.48           |
| 3  | 1.09273 | 0.04371         | 0.56822 | 0.52           |
| 4  | 1.12551 | 0.04502         | 0.63028 | 0.56           |
| 5  | 1.15927 | 0.04637         | 0.69556 | 0.6            |
| 6  | 1.19405 | 0.04776         | 0.76419 | 0.64           |
| 7  | 1.22987 | 0.04919         | 0.83631 | 0.68           |
| 8  | 1.26677 | 0.05067         | 0.91207 | 0.72           |
| 9  | 1.30477 | 0.05219         | 0.99163 | 0.76           |
| 10 | 1.34392 | 0.05376         | 1.07513 | 0.8            |

b. Suppose the real interest rate increases to 5%, but everything else remains as in part (a). Compute the debt-to-GDP ratio in 10 years. (5 pts)

Answer:

|    | GDP     | primary deficit | debt    | debt/GDP ratio |
|----|---------|-----------------|---------|----------------|
| 0  | 1       | 0.04000         | 0.40000 | 0.4            |
| 1  | 1.03    | 0.04120         | 0.46120 | 0.44776699     |
| 2  | 1.0609  | 0.04244         | 0.52670 | 0.496461495    |
| 3  | 1.09273 | 0.04371         | 0.59674 | 0.546101524    |
| 4  | 1.12551 | 0.04502         | 0.67160 | 0.596705437    |
| 5  | 1.15927 | 0.04637         | 0.75155 | 0.64829195     |
| 6  | 1.19405 | 0.04776         | 0.83689 | 0.700880144    |
| 7  | 1.22987 | 0.04919         | 0.92793 | 0.754489467    |
| 8  | 1.26677 | 0.05067         | 1.02499 | 0.809139748    |
| 9  | 1.30477 | 0.05219         | 1.12843 | 0.864851199    |
| 10 | 1.34392 | 0.05376         | 1.23861 | 0.921644426    |

c. Suppose the normal growth rate falls to 1%, and the economy grows at the normal growth rate each year. Everything else remains as in part (a). Calculate the debt-to-GDP ratio in 10 years. Compare your answer to part (b). (5 pts)

Answer:

|    | GDP     | primary deficit | debt    | debt/GDP ratio |
|----|---------|-----------------|---------|----------------|
| 0  | 1       | 0.04000         | 0.40000 | 0.4            |
| 1  | 1.01    | 0.04040         | 0.45240 | 0.447920792    |
| 2  | 1.0201  | 0.04080         | 0.50678 | 0.496790511    |
| 3  | 1.0303  | 0.04121         | 0.56319 | 0.546627947    |
| 4  | 1.0406  | 0.04162         | 0.62171 | 0.597452262    |
| 5  | 1.05101 | 0.04204         | 0.68240 | 0.649283       |
| 6  | 1.06152 | 0.04246         | 0.74534 | 0.702140089    |
| 7  | 1.07214 | 0.04289         | 0.81058 | 0.756043853    |
| 8  | 1.08286 | 0.04331         | 0.87821 | 0.811015019    |
| 9  | 1.09369 | 0.04375         | 0.94831 | 0.867074722    |
| 10 | 1.10462 | 0.04418         | 1.02094 | 0.924244519    |

d. Return to the assumptions of part (a). Suppose policy makers decide that a debt-to-GDP ratio of more than 50% is dangerous. Verify that reducing

the primary deficit to 1% immediately, and that maintaining this deficit for 10 years, will produce a debt-to-GDP ratio of 50% in 10 years. (5 pts)

Answer:

|    | GDP     | primary deficit | debt    | debt/GDP ratio |
|----|---------|-----------------|---------|----------------|
| 0  | 1       | 0.01000         | 0.40000 | 0.4            |
| 1  | 1.03    | 0.01030         | 0.42230 | 0.41           |
| 2  | 1.0609  | 0.01061         | 0.44558 | 0.42           |
| 3  | 1.09273 | 0.01093         | 0.46987 | 0.43           |
| 4  | 1.12551 | 0.01126         | 0.49522 | 0.44           |
| 5  | 1.15927 | 0.01159         | 0.52167 | 0.45           |
| 6  | 1.19405 | 0.01194         | 0.54926 | 0.46           |
| 7  | 1.22987 | 0.01230         | 0.57804 | 0.47           |
| 8  | 1.26677 | 0.01267         | 0.60805 | 0.48           |
| 9  | 1.30477 | 0.01305         | 0.63934 | 0.49           |
| 10 | 1.34392 | 0.01344         | 0.67196 | 0.5            |

#### 4 Time Inconsistency and Monetary Policy (30 pts)

Assume the Philips curve is given by

$$y = y_n + b(\pi - \pi^e)$$

The central bank can control inflation directly. The loss function facing the central bank is

$$L = \frac{1}{2}[a\pi^2 + (y - y_n - m)^2]$$

with  $a$  and  $b$  being positive parameters.

a). Suppose that the central bank can fully commit to a zero inflation, i.e.  $\pi = 0$ . As a result, agents' expected inflation is also fixed at zero, i.e.  $\pi^e = 0$ . What is the value of loss function? Is this optimal? (10 pts)

Answer:

When both  $\pi$  and  $\pi^e$  are 0, according to the Phillips curve,  $y = y_n$ .

The value of loss function is equal to  $\frac{1}{2}m^2$ . It is optimal since committing to any non-zero inflation would only increase the value of the loss function.

b). Suppose now that the central bank is not credible, so it cannot make a commitment about future inflation. This implies that central bank cannot fix agents' expected inflation at 0. Suppose that the agents have rational expectations, that is, their expected inflation is exactly equal to actual inflation. What is the optimal choice of inflation in this case? What is the value of the loss function? Compare it to the one you obtained in part (a), which one is larger? (10 pts)

Answer:

The central bank takes the Phillips curve as given, and choose inflation and output to minimize the loss function. The problem can be written as

$$\begin{aligned} \min_{\pi, y} & \frac{1}{2}[a\pi^2 + (y - y_n - m)^2] \\ \text{s.t.} & \\ y &= y_n + b(\pi - \pi^e) \end{aligned}$$

Substituting the Phillips curve into the objective function,

$$\min_{\pi} \frac{1}{2}[a\pi^2 + [b(\pi - \pi^e) - m]^2]$$

Taking  $\pi^e$  as given, the first order condition with respect to  $\pi$  is

$$a\pi + b^2(\pi - \pi^e) - bm = 0$$

thus when  $\pi = \pi^e$  (rational expectation), the first order condition requires

$$\pi = \frac{bm}{a}$$

Thus the loss function is

$$L = \frac{1}{2}\left(\frac{b^2}{a} + 1\right)m^2$$

which is greater than the loss function under full commitment.

c). Consider the optimal inflation with commitment you obtained in part (a). Given that agents' inflation expectation is fixed at zero, does central bank has the incentive to deviate from this plan? (10 pts)

Answer:

Given  $\pi^e = 0$ , the central bank's problem is

$$\begin{aligned} \min_{\pi, y} & \frac{1}{2}[a\pi^2 + (y - y_n - m)^2] \\ \text{s.t.} & \\ y &= y_n + b(\pi - \pi^e) \\ \pi^e &= 0 \end{aligned}$$

Substituting the constraints into the objective function,

$$\min_{\pi} \frac{1}{2} [a\pi^2 + (b\pi - m)^2]$$

Take the first order condition

$$a\pi + b^2\pi - bm = 0$$

Hence,

$$\pi = \frac{bm}{a + b^2}$$

The loss function's value is

$$L = \frac{m^2}{2} \frac{a}{a + b^2} < \frac{m^2}{2}$$

Hence, the central bank has incentive to deviate from the zero inflation commitment.

MIT OpenCourseWare  
<http://ocw.mit.edu>

14.02 Principles of Macroeconomics  
Spring 2014

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.