

# Time Inconsistency and the Inflation Bias

Notes for 14.02 Spring 2014 \*

April 29 2014

## Abstract

Start from the Phillips curve

$$y = y_n + b(\pi - \pi^e)$$

where  $b$  is a positive parameter. Assume the central bank can directly control inflation,  $\pi$ . It decides the optimal inflation rate by minimizing a quadratic loss function defined in terms of deviations of inflation from a target  $\pi^*$  and of output,  $y$ , also from a target, which we assume to be  $k$  times  $y_n$ , the level of output in the *medium run* equilibrium. This loss function represents society's well being: the smaller the loss, the better off is society

$$L = 1/2 \left[ a(\pi - \pi^*)^2 + (y - ky_n)^2 \right]$$

with  $a$  a positive parameter that describes the relative weight the central bank attaches to deviations from its two objectives.  $k > 1$ : the central bank wishes to keep  $y$  above  $y_n$  <sup>1</sup>. As we shall see it is precisely this incentive that creates an inflationary bias.

Replacing the Phillips curve in the central bank loss function and assuming for simplicity  $\pi^* = 0$  <sup>2</sup>.

$$L = 1/2 \left\{ [a\pi^2] + [(1 - k)y_n + b(\pi - \pi^e)]^2 \right\}$$

We now compute three different solutions to this model of output and inflation.

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\*These notes are an extension (not a substitute) of Chapter 24.

<sup>1</sup>Why would the central bank aim at a level of  $y > y_n$ ? It might want to do so if it thinks that some inefficiency, for instance imperfect competition among firms, depresses output in the medium run equilibrium. If this happens the obvious question is why wouldn't you want to address such inefficiencies directly (e.g. using anti-trust policy), rather than through monetary policy. Here we assume that the central bank takes upon itself to address the effects of such inefficiencies.

<sup>2</sup>The level of the inflation target,  $\pi^*$ , is in general irrelevant. It becomes relevant if we worry about the possibility of the economy being stuck at the Zero Lower Bound, that is when the nominal interest falls to zero and can no longer be reduced. Since the nominal interest rate is equal to the real interest rate plus expected inflation, the higher expected inflation – which is equal to  $\pi^*$  in the medium run – the higher the medium run nominal rate and thus the less likely it will fall to zero.

## 1. Discretionary solution

To determine the *optimal inflation rate* the central bank will choose we take the partial derivative of  $L$  with respect to  $\pi$  and set it equal to zero.  $\max_{\{\pi\}} L$  yields

$$\frac{dL}{d\pi} = a\pi + b[(1 - k)y_n + b(\pi - \pi^e)] = 0$$

from which the optimal inflation rate — for any given level of  $\pi^e$  — is

$$\pi^D = \frac{b}{a + b^2} [b\pi^e + (k - 1)y_n]$$

Assume, for example, for  $\pi^e = 0$ , then

$$\pi^D = \frac{b}{a + b^2} (k - 1)y_n > \pi^e = 0$$

$$y^D = \frac{a + kb^2}{a + b^2} y_n$$

$$\text{where } y_n < y^D < ky_n$$

and

$$L^D = (k - 1)^2 y_n^2 \frac{a}{a + b^2}$$

We identify this solution with the superscript  $D$  to indicate that this is a *discretionary* solution, that is a solution in which the central bank does not commit to keeping  $\pi$  at any particular level: it freely chooses the level of  $\pi$  that minimizes  $L$  *once* private sector expectations have been set — for example, once nominal wage contracts have been signed based on a given  $\pi^e$ .

In the discretionary solution the central bank pays a cost in terms of inflation, which end up above zero, and gains something in terms of output which is higher than  $y_n$ , though not as high as  $ky_n$ .

The *discretionary* solution is not an equilibrium because  $\pi^e \neq \pi^D$ . In other words, it is not an equilibrium because the private sector is fooled. For instance they expect  $\pi^e = 0$  while instead it turns out that the central bank — exploiting the fact that once the private sector chooses  $\pi^e$  it is stuck with it, for instance because nominal wage contracts are fixed for some time — sets  $\pi^D = \frac{b}{a + b^2} (k - 1)y_n > 0$ .

## 2. Rational expectations

We now compute the *rational expectations equilibrium*, that is a solution in which  $\pi = \pi^e$ . Under the assumption of rational expectations the private sector forms its expectations taking into account the fact that the central bank will respond to any value of  $\pi^e$  they choose, setting

$$\pi^D = \frac{b}{a + b^2} [b\pi^e + (k - 1)y_n]$$

Thus they will choose  $\pi^e = \pi^D$ , and that the rational expectations value of  $\pi^e$  will be

$$\pi_{RE} = \pi_{RE}^e = \frac{b}{a} (k - 1) y_n$$

In the rational expectations equilibrium

$$y_{RE} = y_n$$

and the loss is

$$L_{RE} = (k - 1)^2 \frac{a + b^2}{a} y_n^2 > L^D$$

### 3. Commitment

Assume the central bank commits to keep  $\pi = \pi^* = 0$  and not to move  $\pi$  after inflation expectations have been set. In this case

$$\begin{aligned} \pi^C &= \pi^e = 0 \\ y^C &= y_n \\ L^C &= (k - 1)^2 y_n^2 > L^D \end{aligned}$$

#### Time Inconsistency

Note that  $L^C > L^D$ . The central bank's commitment not to deviate from  $\pi = 0$  once expectations have been set (based on the announcement of  $\pi = 0$ ), is *time inconsistent*: by renegeing on its promise the central bank would reduce the loss, i.e. would make everyone better off.

The bottom line is that neither  $\pi^D$  nor  $\pi^C$  are equilibria and the only equilibrium is the rational expectation equilibrium  $\pi_{RE}$  with  $y_{RE} = y_n$  and

$$L^{RE} > L^C > L^D$$

Figure 1 shows the various solutions graphically.

Society would be better off if the central bank was able to commit to  $\pi = 0$  (notice that this holds for any inflation target  $\pi^*$ , not only for  $\pi = 0$ ) and were prevented from re-optimizing once inflation expectations are set. But we've just seen that commitment is time inconsistent<sup>3</sup>. So what can be done?

#### Tying the hands of the central bank

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<sup>3</sup>You will recognize this problem as a manifestation of the *prisoner dilemma* studied in game theory.

One solution consists in tying the hands of the central bank limiting its discretion. For instance writing in the Constitution that the central bank should only care about inflation, that is minimize

$$L = 1/2 (\pi - \pi^*)^2$$

Where the only solution is  $\pi = \pi^*$ . This is in fact what the statutes of many central banks say (*e.g.* the European Central Bank, the Swedish Riksbank, the central banks of Norway, Australia and New Zealand) but not the US Federal Reserve. The law that created the Fed — the Federal Reserve Act of 1913 — says it should care about both inflation and output fluctuations, that is minimize

$$L = 1/2 \left[ a (\pi - \pi^*)^2 + (y - y_n)^2 \right]$$

### How do central banks behave in practice? The Taylor rule

Go back to the Phillips curve and re-write it as

$$\pi = \pi^e + \beta (y - y_n)$$

or

$$\pi - \pi^e = \beta (y - y_n)$$

If the central bank was able to keep  $\pi = \pi^e = \pi^*$ , then  $y = y_n$ . The central bank gives up the attempt to move  $y$  away from  $y_n$  (i.e. it sets  $k = 0$ ), but at least achieves its inflation target. This is probably the reasonable thing to do. The central bank has only one instrument, the interest rate: with one instrument it can hardly achieve two objectives,  $\pi = \pi^*$  and  $y = ky_n$ .

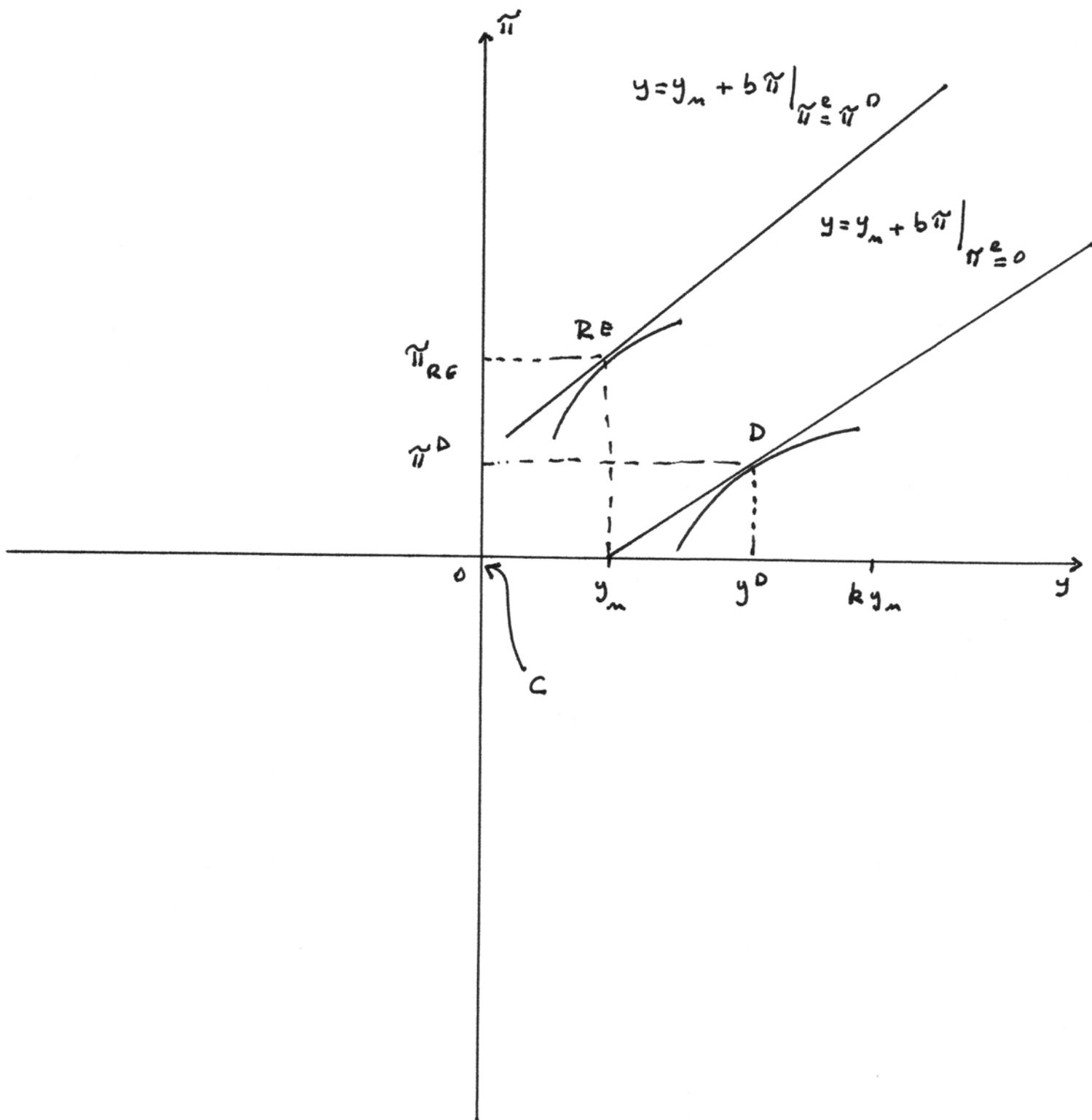
Assume you start from  $\pi = \pi^e = \pi^*$ , and  $y = y_n$ . Now assume that some shock moves  $\pi^e$  away from  $\pi^*$ , e.g.  $\pi^e > \pi^*$ . If the central bank maintains  $\pi = \pi^*$ , since  $\pi - \pi^e = \beta (y - y_n)$ ,  $y < y_n$ , output will fall below the natural rate. This may not be optimal, i.e. it may be better to let  $\pi$  deviate for some time from  $\pi^*$  and limit the fall in output below  $y_n$ . This can be done setting the interest rate following this rule (often referred to as the Taylor Rule from Stanford university economist John Taylor)

$$i = i^* + \gamma (\pi^e - \pi^*) + \delta (y - y_n)$$

where the parameters  $\gamma$  and  $\delta$  describe the relative weights the central bank assigns to deviations of output and inflation from their targets and  $i^* = r^* + \pi^*$  is the nominal interest rate target for a target real rate  $r^*$ .

This is how some central banks, e.g. the Federal Reserve, set monetary policy.

Graphical illustration of the C and D solutions and of the RE equilibrium



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