14.03 Fall 2010

Problem Set 1

Professor: David Autor

Late problem sets are not accepted

1 Implicit Function Theorem and Envelope Theorem

Consider the function:

$$y = f(x; a) = 3ax - 8x^2$$

where a is a parameter.

1. Find the critical point of this function.
2. Is this point a maximum, a minimum, or neither? (Show why)
3. Solve for $x^*(a)$, the maximizer of the function $f(x; a)$ .Also find $y^*(a)$, the maximized value of y as a function of a.
4. Find $\frac{dx^*}{da}$ and $\frac{dy^*}{da}$.
5. Now use the FOC from 1. and the implicit function theorem to find $\frac{dx^*}{da}$.
6. Use the envelope theorem to find $\frac{dy^*}{da}$. Why does this theorem allow you to simplify your calculations with respect to point 4.?

2 Constrained Optimization

Consider the problem

$$\begin{aligned} z &= x^{\frac{1}{3}} y^{\frac{2}{3}} \rightarrow \max_{x,y} \\ s.t.\ 3x + y &= 9 \end{aligned}$$

1. Set up the lagrangian and find the FOCs.
2. Find z^* the maximum, x^* and y^* where the maximum is achieved, and λ the Lagrange multiplier. What is the interpretation of the Lagrange multiplier in this problem?
3. Set up the dual problem, i.e., minimize the primal constraint function subject to the primal objective function constrained to be equal to the value of z^* obtained in 2.
4. Set up the dual Lagranigan and find the FOCs.
5. Find x^D and y^D where the maximum is achieved and λ^D the Lagrange multiplier in the dual problem. What is the interpretation of the Lagrange multiplier in the dual problem?
6. What is the relationship between λ and λ^D? Why?

3 Minimum Wage

Consider a market where a good is produced using only labor. The production function is

$$Y = -2L^2 + 6L,$$

where Y is the output of a good and L is the quantity of labor used. The price of a good is $p = 1$. The labor supply, as a function of the wage is the following

$$L^S = w,$$

where w is the wage per unit of labor.

1. Find the equilibrium wages and employment (w^C, L^C) that would prevail if the market for labor was competitive. [Remember that a competitive firm takes the

wage as given – that is, it assumes that the quantity of workers that it hires has no effect on the price of the next worker. (Of course, the equilibrium wage must equate demand and supply)].

2. Find the equilibrium wages and employment (w^M, L^M) that would prevail if the market was dominated by a single producer that acts as a monopsonist in the labor market. [In this case, the firm accounts for the fact that the quantity of labor it hires affects the market wage—it is not a price taker. Also, recall that the marginal revenue product of labor MRPL is the price of output (Y) times the marginal product of labor.]

3. Show the marginal cost curve for labor faced by the monopsonist and draw a graph that shows the two equilibria you found.

4. Compare the two equilibria and discuss the reasons why they differ.

5. Suppose Congress agrees to suddenly raise the minimum wage to $w_{\text{min}} = 1.4$. What would you expect to observe after the introduction of the minimum wage under each of the two market structures (competitive and monopsonistic)? [Provide a mathematical answer.]

6. What if the minimum wage were raised to $w_{\text{min}} = 2.5$? Explain.

7. Suppose that the demand for a final good is given by

$$Y^D = \begin{cases} \frac{36}{5}\left(1 - \left(p - \frac{1}{3}\right)^2\right) & \text{if } p > \frac{1}{3} \\ \frac{36}{5} & \text{if } p \leq \frac{1}{3} \end{cases}.$$

Check that (w^M, L^M) that you found in 2. is an equilibrium that would prevail if the market was dominated by a single producer that acts as a monopsonist on the labor market but takes the price of a final good p as given.

8. Now consider a market dominated by a single producer that acts as a monopsonist on the labor market and as a monopolist on the product market. Hence is able to both set prices for the final good and the wages it hires labour at. Do you think this will lead to higher or lower labour levels compared to part (2). Explain. *Note, you are not expected to show this algebraically*

4 Causal inference

Twelve months ago, the state of Massapequa expanded its free health insurance program for poor families with small children. To be eligible for the insurance, households must have an income below $30,000 per year and must have children under the age of 12. The government now wants to assess the effect of this expansion in health insurance on the health status of the people it aimed to help.

Let $X_i \in \{0,1\}$ be an indicator variable equal to 0 if person i did not receive insurance through this program and 1 if they did. Let Y_{it} be the variable of interest, which is equal to 1 if person i is healthy (has no chronic conditions) and 0 otherwise. Define $T^* = E\left[Y_1 - Y_0 | X = 1\right]$ as the expected improvement in health status when a person is on the program compared to when they are not (we condition on $X = 1$ since we can only hope to learn about the causal effect of treatment on those who receive treatment).

For each of the following scenarios: State the assumptions required to obtain a valid estimate of T^*, i.e., the average treatment effect for the treated population. Are these assumptions likely to be satisfied? How would you expect the estimate of T^* (call this $\hat{T}$) to be affected if these assumptions are violated (e.g. will $\hat{T}$ be unbiased, biased upward, or biased downward relative to T^*, or is this indeterminate), and why? (Note that upward biased means that $E\left[\hat{T}\right] > E\left[T^*\right]$).

1. The state has access to the complete health records of all residents of the state. They compare the percent of people without chronic conditions among people who received insurance through the program and people who are currently uninsured. Denote these averages by $\hat{Y}_{It}$ and $\hat{Y}_{NIt}$, where I and NI denote the insured and not-insured populations, and t is the date. They compare these averages to determine by how much the health status of people on the program has improved, that is $\hat{T} = \hat{Y}_{It} - \hat{Y}_{NIt}$

2. The state also has data on the health status in the prior (and current) year of the population that became insured this year. Denote these health status outcomes by $\hat{Y}_{It-1}$ and $\hat{Y}_{It}$. They compare the health status of people that received insurance before and after they were enrolled to determine how much their health status has improved, that is $\hat{T} = \hat{Y}_{It} - \hat{Y}_{It-1}$.

3. How would you use the data in parts 1 and 2 to construct a difference-in-difference estimator for the causal effect of health insurance on health status? Under what

assumptions does this provide an unbiased estimate of T^*?

4. Researchers find out that due to limited funds only half of the eligible families received insurance in the first year. The state used a random lottery to decide who would get insurance. All the families that satisfied the eligibility criteria were automatically signed up for the lottery. Half of the families won the lottery and received insurance and half did not. How would you use this information to estimate the causal effect of interest?

5. It turns out that after winning the lottery, to actually receive insurance, families had to submit some paperwork to register. However, some of the families who won the lottery never submitted the required paperwork. In the language of randomized experiments we say that they did not 'comply' with the treatment. Since they never submitted the paperwork, they did not receive health insurance.

   To estimate the effect of having health insurance on health, the researchers pool together all of the people that did not receive insurance (the ones who did not submit the paperwork and the ones who did not win the lottery) and find that 12% of them do not have any major health problems, compared to 10% of the people that did receive insurance (won the lottery and submitted the paperwork). They conclude that the expansion in insurance lowered the rate of health problems by 2 percentage points. Under what assumptions would this be a valid estimate? Are these assumptions likely to hold? How do you think the results would be biased in this case?

5 Minimum wage

In class we discussed the main research design in the Card and Krueger (1994) paper, which compares the changes in employment at restaurants in New Jersey and Pennsylvania to estimate the causal effect of an increase in minimum wage on employment. In a second research design, the authors compare changes in employment at restaurants with high and low initial wages in New Jersey. The following question is loosely based on this second research design.

Let ΔE_i be the change in employment from survey wave 1 to wave 2 at fast-food store i, and W_{1i} the wage in survey wave 1 at fast-food restaurant i. Denote by $\Delta E_{NJ,high}$

and $\Delta E_{NJ,low}$ the average change in employment at New Jersey restaurants with high initial wages $W_{1i} > \$5.05$ and low initial wages $W_{1i} < \$5.05$ respectively. Recall that the minimum wage in New Jersey restaurants was increased from \$4.25 per hour to \$5.05 per hour.

1. What would you expect the causal effect of the increase in minimum wage on employment at restaurants in New Jersey with initial wages above \$5.05 to be? Discuss this in the context of the theoretical models you saw in class (competitive and monopsonistic).

2. Under what assumptions would $\Delta E_{NJ,low} - \Delta E_{NJ,high}$ be an unbiased estimate of the causal effect (T^*) of an increase in the minimum wage on employment at restaurants that initially had wages below the new minimum (i.e, $W_{1i} < \$5.05$)? Are these assumptions for causal estimation likely to hold? (Hint: think about the time effect).

3. Assume that the contrast $\Delta E_{PA,low} - \Delta E_{PA,high} \neq 0$ (that is, it is non-zero) for fast food restaurants in Pennsylvania (which of course did not experience a minimum wage increase). How would you interpret this finding? Does this make it more or less likely that $\Delta E_{NJ,low} - \Delta E_{NJ,high}$ (the quantity in part (2)) is an unbiased estimate of T^*?

4. Under what assumptions would $(\Delta E_{NJ,low} - \Delta E_{NJ,high}) - (\Delta E_{PA,low} - \Delta E_{PA,high})$ provide an unbiased estimate of T^*? Explain how this difference-in-difference estimate potentially addresses your concerns in part 2.