

# Lecture Note 7: Revealed Preference and Consumer Welfare

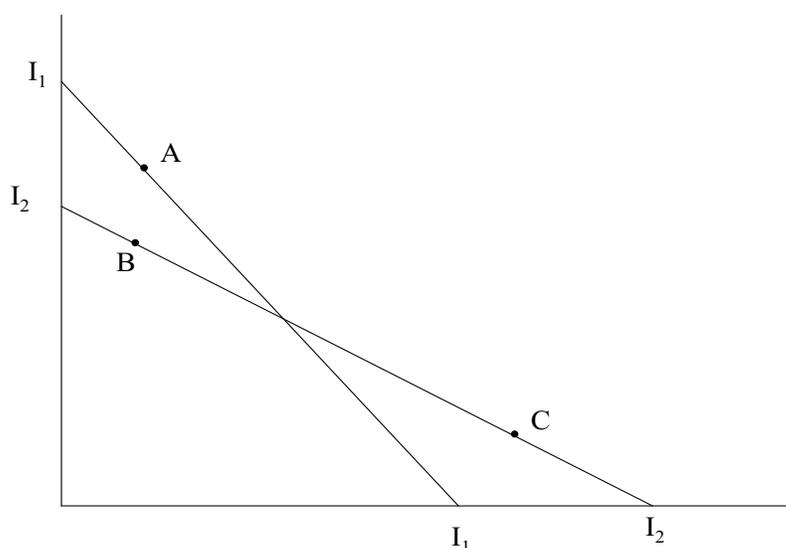
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# 1 Revealed Preference and Consumer Welfare

- We have to date been applying an axiomatic approach to characterizing consumer choice, based on the five axioms given in Lecture 3.
- But there is an approach to assessing utility that requires even fewer assumptions and nevertheless gives strong results. This approach is called *Revealed Preference*.
- Consider the figure below, where the consumer sequentially faces two budget sets,  $I_1 - I_1$  and  $I_2 - I_2$ . Point  $A$  on  $I_1 - I_1$  is initially chosen. This point is said to be “revealed preferred” to all other feasible points inside of the budget set.

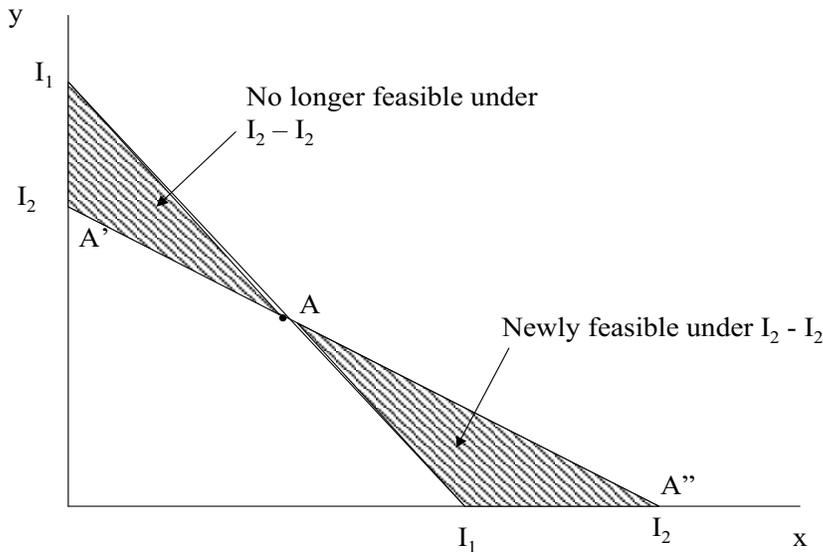
8#1



- Now the consumer is faced with  $I_2 - I_2$ .
  - If the consumer chooses point  $B$ , are they better or worse off? *Worse off*, because point  $A$  was revealed preferred to point  $B$  under the initial choice conditions.
  - If they choose point  $C$ , are they better or worse off? The axiomatic approach would tend to suggest they are better off because unless the indifference curve tangent to point  $A$  has an extremely shallow slope, point  $C$  would probably put them on a higher indifference curve.
  - But under Revealed Preference, the answer is ambiguous. The reason is that we do not have any revealed preference information on whether  $A$  is preferred to  $C$  or vice versa—these choices were never available simultaneously.

- Take a second example (below)

8#2



Here the second budget set rotates through the originally chosen point  $A$  on the first budget set. We do not know the consumer's new choice. Is the consumer better off, worse off, or can't we say?

- We know that point  $A$  r.p.  $(A, A')$
- We do not know—but it is possible—that a point on  $(A, A'')$  is preferred to  $A$ .
- We say that the consumer is “weakly better off.”

**Definition 1** *Weak Axiom of Revealed Preference:* If  $A, B$  feasible and  $A$  chosen, then at any prices and income where  $A, B$  are feasible, the consumer will choose  $A$  over  $B$ .

This axiom says two things:

1. People choose what they prefer.
2. Preferences are *consistent*. Therefore, a single observed choice reveals a stable preference.

There is also a stronger form of this axiom.

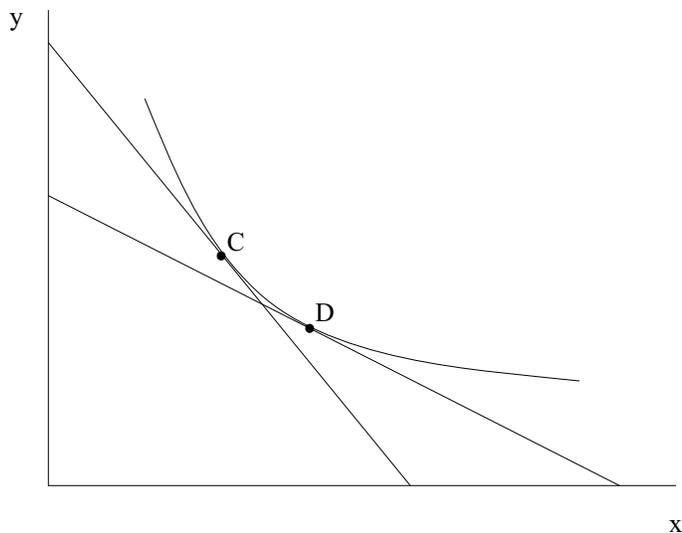
## 1.1 The power of WARP

### 1.1.1 Demonstration 1

- The result that  $\frac{\partial X}{\partial P_x}|_{u=u_0} < 0$  (i.e., the compensated demand curve is always downward sloping) depends on an untested assumption about diminishing MRS, giving rise to indifference curves that are bowed inward towards the origin.
- Can we obtain the same result using only WARP?
- Suppose two points  $C, D$  on intersecting budget sets.
- Assume that these points are indifferent for consumer utility (see Figure 8#3). That is, the consumer has told us that he or she is indifferent.

$$C = (X_c, Y_c) \sim D = (X_d, Y_d).$$

8#3



- Note that the ‘indifference curve’ drawn here is simply meant to represent the fact that the consumer *says* she is indifferent. There is *no notion of indifference curves* in the Revealed Preference approach.

- Since  $C, D$  do *not* lie in the same budget sets, when  $C$  is available,  $D$  is not and vice versa.
- Since  $C \sim D$ , it must be true by WARP that

$$\begin{aligned} P_x^c X_c + P_y^c Y_c &\leq P_x^c X_d + P_y^c Y_d \text{ when } C \text{ is chosen,} \\ P_x^d X_d + P_y^d Y_d &\leq P_x^d X_c + P_y^d Y_c \text{ when } D \text{ is chosen.} \end{aligned}$$

- Rearranging, we get:

$$\begin{aligned} P_x^c(X_c - X_d) + P_y^c(Y_c - Y_d) &\leq 0, \\ P_x^d(X_d - X_c) + P_y^d(Y_d - Y_c) &\leq 0. \end{aligned}$$

which simply says that at prices where  $C$  is purchased,  $D$  must have been at least as expensive as  $C$  (or  $C$  would have been purchased), and at prices where  $D$  is purchased,  $C$  must have been at least as expensive as  $D$  (or  $D$  would have been purchased).

- Combining, we get (we add the two inequalities - remember if something is smaller than or equal to zero, the combination of the two must also be smaller than or equal to zero)

$$(P_x^c - P_x^d)(X_c - X_d) + (P_y^c - P_y^d)(Y_c - Y_d) \leq 0. \quad (1)$$

- Now, consider a case where only the price of  $X$ ,  $(P_x)$  changes and assume that  $P_y^c = P_y^d$ . Using (1), this gives

$$(P_x^c - P_x^d)(X_c - X_d) \leq 0,$$

which in calculus terms is equivalent to:

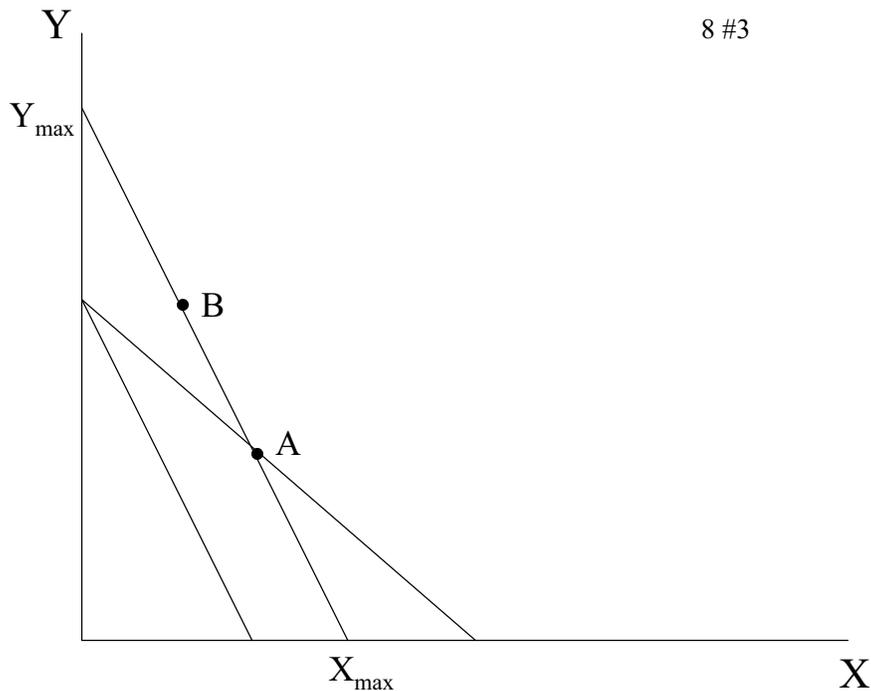
$$\frac{\partial X}{\partial P_x} \Big|_{u=u_0} \leq 0.$$

(Remember that  $C \sim D$ , so we are holding utility constant.)

- *So, WARP is sufficient to establish weakly downward sloping compensated demand curves. [Why Compensated? Because utility is held constant in this example since  $C \sim D$ .]*
- The *entire* idea of revealed preference is simply by using the weak notion of “choosing what you prefer,” you get strong rationality properties, including:
  - Weakly downward sloping demand curves.
  - Only relative prices matter (as can be seen in the example above).
- We therefore don’t have to make strong assumptions about diminishing MRS to get strong predictions about “rational” behavior.

### 1.1.2 Demonstration 2

- Here's a second proof of the above that does not invoke 'indifference.'
- Without talking about indifference curves, we can't really hold  $U$  constant. But we can still talk about compensated demand. Suppose that the consumer is initially buying some bundle  $A$ . The price of  $X$  goes up, and we give the consumer just enough extra income to make  $A$  affordable again (see figure below).



- The new budget line is  $X_{\max}Y_{\max}$ . Is it possible that the consumer would choose a point on this budget line between  $X_{\max}$  and  $A$ ?
- No. These points were all available to the consumer at the original prices and income, and the consumer chose  $A$  instead.
- Under WARP, the only points on the new budget line that the consumer would be expected to choose are between  $A$  and  $Y_{\max}$ , since these were not available before. These points all have the consumer buying the *same or less*  $X$  than she bought at  $A$  (when the price of  $X$  was lower).
- Note: the potential weakness of this proof is that the Axiomatic approach to consumer

theory would suggest that the consumer is strictly better off along the new budget set than the old one (given that  $A$  is available, she cannot be worse off; given reasonable preferences, she is likely on a higher indifference curve). Thus, this proof does not strictly hold utility constant, which is required for the definition of compensated demand.

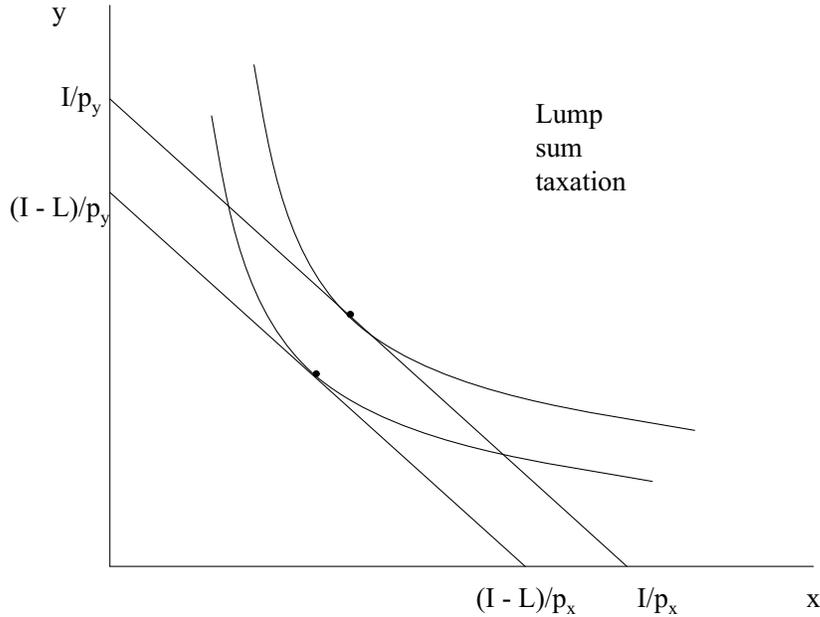
## 1.2 The Strong Axiom of Revealed Preference (SARP)

**Definition 2** *Strong Axiom of Revealed Preference (SARP):* If commodity bundle 0 is revealed preferred to bundle 1 and bundle 1 is r.p. to bundle 2 and bundle 2 is r.p. to bundle 3... and bundle  $k - 1$  is r.p. to bundle  $k$ , then bundle  $k$  cannot be r.p. to bundle 0.

SARP is simply WARP with an added transitivity assumption. But this places much greater strictures on behavior.

## 2 Using WARP to evaluate the consequences of taxation

- For many reasons, governments need to tax:
  - Pay for public goods: Defense, law enforcement, regulatory agencies.
  - Transfer income–social insurance.
  - Correct externalities (pollution, ‘sins’).
- Are there better and worse ways to tax?
- Let’s compare two types of taxes:
  - A lump-sum tax: reduces the consumer’s budget by  $L$ .
  - A sales tax on a single good: charge tax  $t$  on  $X$  so that  $P_x^t = P_x + t$ .
- Obviously, consumer’s are worse off for being taxed, but can we say anything stronger than that?
- See figure.



- Note the algebra of the lump sum tax:

$$XP_x + YP_y = I, \quad (2)$$

$$X_L P_x + Y_L P_y = I - L, \quad (3)$$

$$(X - X_L)P_x + (Y - Y_L)P_y = L$$

- To compare the lump-sum tax with a revenue equivalent sales tax, we need “revenue equivalence” (i.e., same level of taxes collected).
- Let’s consider a tax of  $t^*$  on purchases of good  $X$ . So, for every  $X$  consumed, the consumer pays  $t^*$  in taxes.
- For  $t^*$  to be revenue-equivalent to  $L$ , the following condition must hold:

$$t^* \cdot d_x(P_x + t^*, P_y, I) = L.$$

In words, the “revenue equivalent” sales tax generates the same total taxation as  $L$  by charging  $t^*$  on each  $X$  purchased.

- To see this, note:

$$X_t(P_x + t^*) + Y_t P_y = I, \quad (4)$$

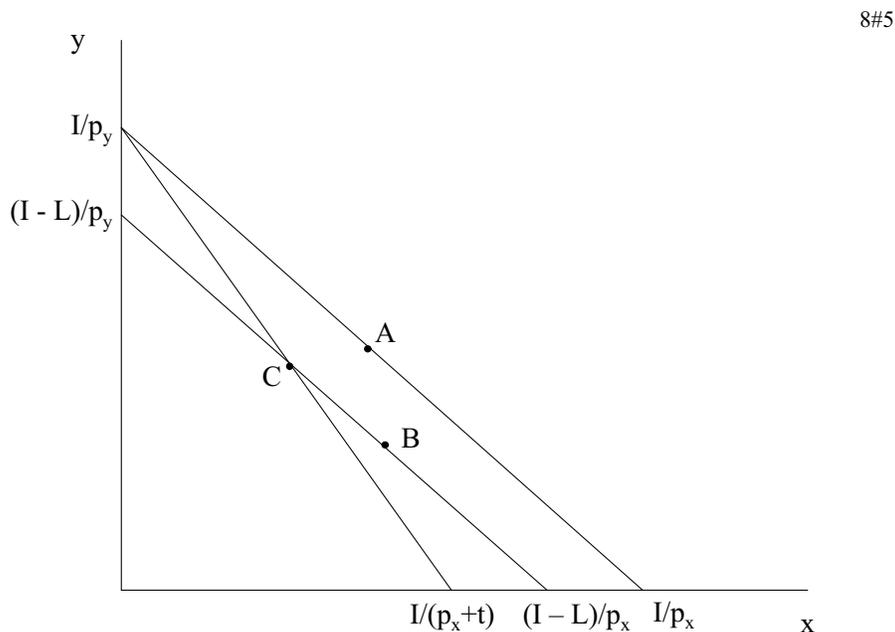
$$I - X_t P_x - Y_t P_y = X_t t^*.$$

- So, for revenue equivalence, we need  $X_t t^* = L$ .
- We know from (2) that the budget set that characterizes the lump-sum tax is given by  $X_L P_x + Y_L P_y = I - L$ .

- So, the revenue equivalent tax *must* also put the consumer on this budget set. Hence,

$$X_t P_x + Y_t P_y = X_L P_x + Y_L P_y.$$

- Graphically, the revenue equivalent tax is the tax that causes the consumer to consume on the Lump-sum budget set. See figure.



- Notice that the exact tax  $t^*$  that solves this problem depends upon consumer preferences. If consumption of  $X$  is highly elastic to the tax (that is, it falls precipitously), we'll need to set  $t$  fairly high to get  $t^* X_t = L$ .
- Since the tax puts the consumer back on the lump-sum budget set, does this imply that she is just as well off under either tax scheme?
- No. By WARP, the consumer is weakly worse off under the sales tax than the lump-sum tax.
- By shifting the price ratio, the tax has caused the consumer to choose a point on the lump-sum budget set that is not the most preferred point on this set. *Tax has distorted the choice.*

- This is a Revealed Preference argument: We know by WARP that  $B$  is the most preferred point on the budget set  $[(I - L) / p_x, (I - L) / p_y]$ . So, if taxation causes the consumer to choose any point other than  $B$  on this budget set, the consumer *must be* at least weakly worse off.
- A powerful and general result: If you must tax, you harm consumers less by simply taking a chunk of their budget than by distorting prices *and* ultimately taxing an identical share of their budget.
- Drawing on the axiomatic approach to consumer utility, what is the exact distortion? In the lump-sum case, it remains true that

$$\frac{U_x}{U_y} = \frac{P_x}{P_y}.$$

- Whereas in the revenue equivalent taxation case, the consumer's 'optimal choice' will satisfy

$$\frac{U_x}{U_y} = \frac{P_x + t^*}{P_y}.$$

- Their consumption choices do not reflect the 'real costs' of goods provided in the market – they are distorted by the tax. There are producers willing to sell  $X$  at price  $P_x$  and consumers who would gladly pay  $P_x$ . But they will not purchase at this price due to the tax. However, consumers and producers can transact for  $Y$  at price  $P_y$ . As a result consumers will under-consume  $X$  and overconsume  $Y$  relative to their real market costs.
- Consider: What would be the efficiency consequences (relative to a revenue equivalent lump sum tax) of charging the same proportional tax on all goods?

## 2.1 Proof of distortionary impact of non-neutral taxation of goods

- Consider (implausibly) a tax that is *fully rebated* to the consumer:

$$t \cdot d_x(P_x + t, P_y, I + Z) = Z. \tag{5}$$

- This tax is revenue neutral for consumer; rebated exactly the amount paid in taxes ( $Z$ ).
- Hence, only effect is to alter the price ratio faced by consumer.
- A critical (but strange) assumption here is that the consumer does not realize that the tax is fully rebated; that is, when the consumer buys  $X$ , she does not consider that she'll get

$Z = tX$  tax rebate in return. If she did realize this, it would make the exercise pointless; the consumer would, in effect, face no real tax on  $X$ . This can be reconciled with the assumption there is a multiple identical individuals with the same preferences, therefore one individual's rebate will be dependant on other people's choices and not his own.

- You can check that the consumer spends the original budget  $I$  by writing:

$$(P_x + t) \cdot d_x(P_x + t, P_y, I + Z) + P_y \cdot d_y(P_x + t, P_y, I + Z) = I + Z.$$

- Subtracting (5) from both sides, we get

$$P_x \cdot d_x(P_x + t, P_y, I + Z) + P_y \cdot d_y(P_x + t, P_y, I + Z) = I.$$

Hence, the consumer is on the original budget set.

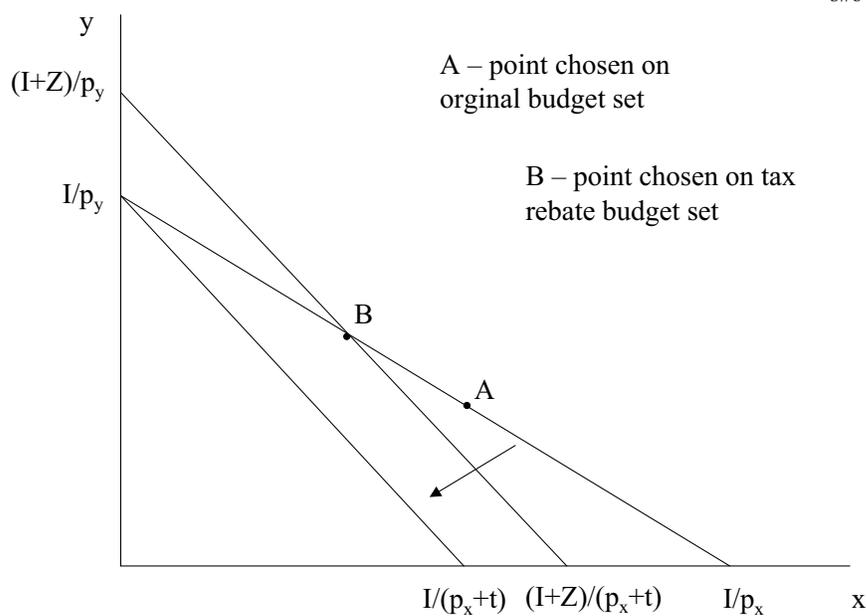
- But as long as the consumer changes the consumption bundle in response to the tax-ratio (i.e., as would occur for any utility function satisfying the standard 5 axioms), then the consumption bundle is 'distorted' by the tax:

$$\begin{aligned} d_x(P_x + t, P_y, I + Z) &\neq d_x(P_x, P_y, I), \\ d_y(P_x + t, P_y, I + Z) &\neq d_y(P_x, P_y, I). \end{aligned}$$

- In words, the consumer will be consuming on a different point on the original budget set  $I$  under the 'taxed' price ratio.
- If so, the consumer is *worse off* by Revealed Preference.
- Hence, the distortion induced by non-neutral taxation is that it causes the consumer to pick a non-preferred point on the 'true' (non-distorted) budget set.
- Note that this argument does not depend on any axioms of utility theory *other than* WARP. The essential point is:
  - We know that the consumer will *have to* pick a point on the original budget set for the tax to be revenue equivalent.
  - But rather than allow the consumer to simply face that budget set and choose the preferred point, we are distorting her behavior by shifting the slope while ultimately placing them back somewhere on the same line.

- If they choose any other point than the preferred point on the un-distorted budget set, they are weakly worse off. (Only weakly because we have no way of knowing whether the consumer was indifferent among multiple points on the original budget set, such as *A* and *B*).

8#6



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