

Lecture Note 16—Adverse Selection, Risk Aversion and Insurance Markets.

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David Autor, Massachusetts Institute of Technology

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1 Adverse Selection, Risk Aversion and Insurance Markets

- Risk is costly to bear (in utility terms). If we can defray risk through market mechanisms, we can potentially make many people better off without making anyone worse off.
- We gave three explanations for why and how insurance markets operate:
 1. Risk pooling—Law of large numbers.
 2. Risk spreading—Social insurance for non-diversifiable risks.
 3. Risk transfer (Lloyds of London)—Trading risk between more and less risk averse entities.
- (Note: risk spreading does not generate Pareto improvements, but it may still be economically efficient.)
- There is an exceedingly strong economic case for many types of insurance. Efficient insurance markets can unequivocally improve social welfare.
- If the economic case for full insurance is so strong, why do we *not* see full insurance for:
 - Health
 - Loss of property: home, car, cash
 - Low wages

- Bad decisions:
 - * Marrying wrong guy/gal
 - * Going to the wrong college
 - * Eating poorly
- Instead, we see:
 - Markets where not everyone is insured (health insurance, life insurance)
 - Incomplete insurance in every market where insurance exists at all:
 - * Deductibles
 - * Caps on coverage
 - * Tightly circumscribed rules (e.g., must install smoke detectors in house, must not smoke to qualify for life insurance).
 - * Coverage denied
 - * Insurance markets that don't exist at all, even for major life risks
 - Low earnings
 - Bad decisions
- Why are insurance markets incomplete?
- Roughly 4 explanations:
 1. Credit constraints: People cannot afford insurance and hence must bear risk. Health insurance could be an example (i.e., if you already know you have an expensive disease, it may be too late to buy insurance).
 2. Non-diversifiable risk cannot be insured, e.g., polar ice cap melts, planet explodes. No way to buy insurance because we all face identical risk simultaneously.
 3. Adverse selection—Individuals' private information about their own 'riskiness' causes insurers not to want to sell policies to people who want to buy them.
 4. Moral hazard ('hidden action')—Once insured, people take risky/costly actions that they otherwise would not. This makes policies prohibitively costly or simply not worth offering (i.e., if I know that I'll drive like an idiot as soon as I have auto insurance, I may not want to be insured).

- The model that we'll develop in this lecture concerns adverse selection in insurance markets. The basic notion of this model is that potential insured consumers may differ in their risk profiles (that is, their expected costs), and, moreover, consumers may have some private information about their 'riskiness' that is unknown to insurance companies (that is, insurance companies may not be able to tell high from low-risk consumers). It turns out that a small degree of private information in insurance markets may be sufficient to cause substantial market failures.

2 The Environment

- Consider an insurance market where each potential insured faces two states of the world.
 1. No accident, in which case wealth is w .
 2. Accident, in which case wealth is $w - d$ (where $d > 0$ stands for damage).
- So, each person's wealth endowment is:

$$W_i = \begin{cases} \Pr(p_i) & w \\ \Pr(1 - p_i) & w - L \end{cases}$$

- If a person is insured, her endowment is changed as follows

$$W_i = \begin{cases} \Pr(p_i) & w - I \\ \Pr(1 - p_i) & w - L - I + B \end{cases}$$

- Here, the parameters (I, B) describes the insurance contract, with I equal to the insurance premium (paid in either state of the world) and B as the benefit paid if there is an accident. We do *not* assume that $B = I/p$. That is, we are not assuming that the insurance is actuarially fair, nor do we assume that the insured can buy any coverage amount, B , that she prefers. These parameters may be set by insurance carriers.
- Denote the probability of an accident as p . An individual will buy insurance if the expected utility of being insured exceeds the expected utility of being non-insured, i.e.,

$$(1 - p) \cdot U(w - I) + p \cdot U(w - L - I + B) > (1 - p) \cdot U(w) + p \cdot U(w - L).$$

- An insurance company will sell a policy if expected profits are non-negative:

$$I - pB \geq 0.$$

- Competition will insure that this equation holds with equality, and hence in equilibrium:

$$I - pB = 0.$$

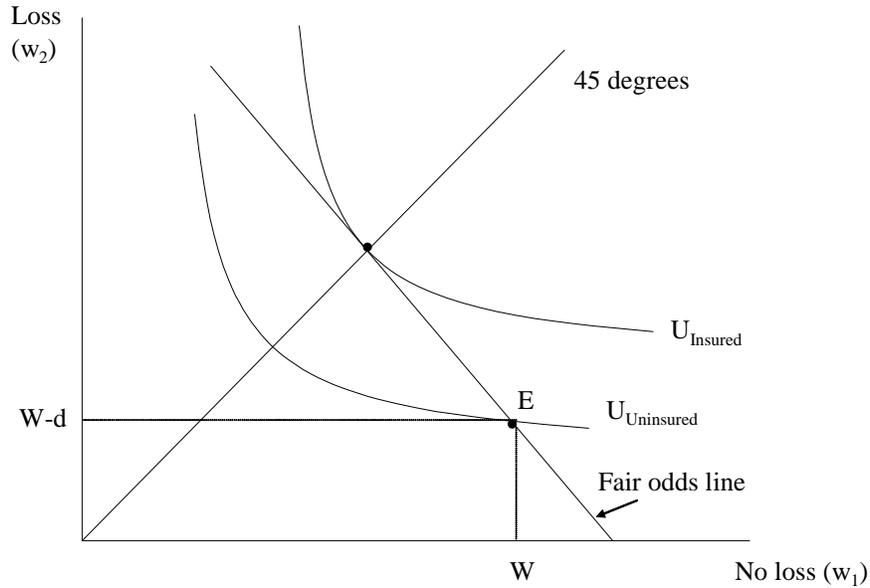
- We now need to define an equilibrium for this model. RS propose the following equilibrium conditions:

1. **No insurance contract makes negative profits (break-even condition).**
2. **No contract outside of the set offered exists that, if offered, would make a non-negative profit.** *If there were a potential contract that could be offered that would be more profitable than the contracts offered in equilibrium, then the current contracts cannot be an equilibrium.*

3 Base case: Homogeneous risk pool [review]

- To fix ideas, it is always useful to start with the simplest case.
- Assume for now that all potential insured have the same probability of loss, $p : p_i = p \forall i$. And note that we have already assumed that all losses are equal to L .
- (Fixing L is without loss of generality. We only need one free parameter here, either p or L . We'll be using p below.)
- See the Figure 1 below, which recaps the state preference diagram from our prior lectures on risk aversion and insurance.
- Note that from the initial endowment $E = (w, w - L)$, the fair odds line extends with slope $\frac{-(1-p)}{p}$, reflecting the odds ratio between the accident and no-accident states.
- As we showed some weeks ago, a risk averse agent (everyone is assumed to be risk averse here) will optimally purchase full insurance if this insurance is actuarially fair. Following from the Von Neumann Morgenstern expected utility property, the highest indifference curve tangent to the fair odds line has slope $\frac{-(1-p)}{p}$ at its point of tangency with the the fair odds line, which is where it intersects the 45^0 line. At this point, wealth is equalized across states. So, the tangency condition is

$$\frac{(1-p)u'(w-I)}{pu'(w-I-L+B)} = \frac{(1-p)}{p} \Rightarrow w-pL = w-pL-L+L.$$



- In this initial case, insurance companies will be willing to sell this policy since they break even.
- This will be an equilibrium since no alternative profit-making policies could potentially be offered.

4 Adding heterogenous risk and private information

- We extend the model to the case with:
 1. Heterogeneity: The loss probability p varies across individuals. Specifically, assume two types of insurance buyers:

h : Probability of loss p_h ,

l : Probability of loss p_l ,

with $p_h > p_l$.

These buyers are otherwise identical in w and the amount of loss L in event of an accident (and their utility functions $u(\cdot)$). Only their odds of loss differ.

2. Private information: Assume that individuals' i know their risk type p_i but this information is not known to insurance companies. (Note: private information without heterogeneity is not meaningful; if everyone is identical, there is no private information.)
- How realistic is the latter assumption? The gist is clearly correct: you know more about your 'riskiness' than your insurance companies. It is this informational advantage that is at the heart of the model. Although the model presents a particularly stark case, the same general results would hold with any degree of informational asymmetry.
 - Given the two risk types $\{h, l\}$ and the asymmetric information about these types, there are two possible classes of equilibria in the model:
 1. **Pooling equilibrium:** All risk types buy the same policy.
 2. **Separating equilibrium:** Each risk type (h, l) buys a different policy.
 - We'll take these possibilities in order.

5 Candidate pooling equilibrium

- In a pooling equilibrium, both risk types buy the same policy.
- The equilibrium construct requires that this policy lie on the *aggregate* fair odds line (so that it earns neither negative nor positive profits).
- Define λ as the proportion of the population that is high risk.
- The expected share of the population experiencing a loss is

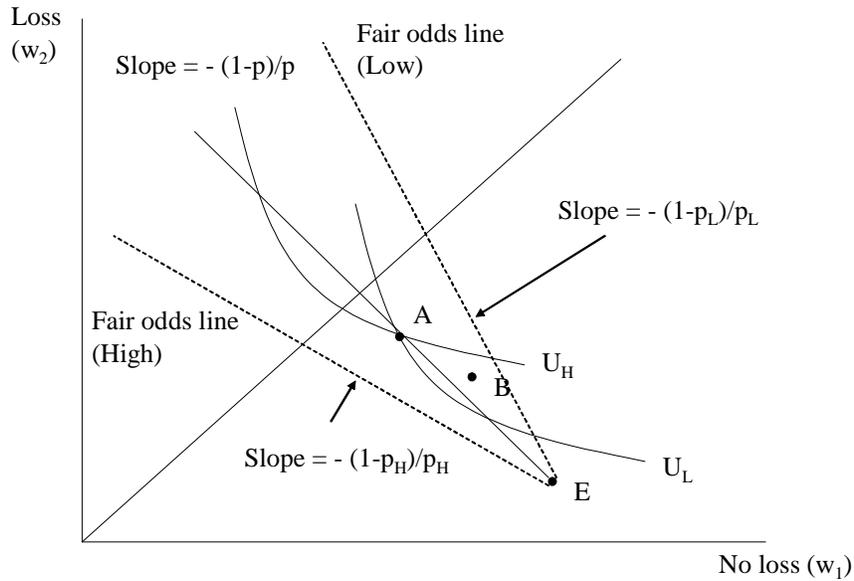
$$\bar{p} \equiv \lambda p_h + (1 - \lambda)p_l.$$

The expected share not experiencing a loss is

$$1 - \bar{p} = 1 - (\lambda p_h + (1 - \lambda)p_l).$$

- The slope of the aggregate fair odds line is

$$-\frac{1 - \bar{p}}{\bar{p}}.$$



- See Figure 2:
- Notice first that the ‘pooling’ policy A must lie on the aggregate fair odds line. If it lay above, it would be unprofitable and so would not exist in equilibrium. If it lay below, it would it would make positive profits and so would not exist in equilibrium.
- Notice 2nd that the figure is drawn with:

$$|MRS_{w_1, w_2}^h| < |MRS_{w_1, w_2}^l|.$$

This is quite important. How do we know it’s true?

- If the pooling policy is actuarially fair (on average):

$$\begin{aligned} w_1 &= w - \bar{p}A, \\ w_2 &= w - \bar{p}A - L + A, \end{aligned}$$

where A is the amount of the payment (technically, A is equal to the difference in vertical height between E and A . We’re just calling this A for notational simplicity.)

- From the VNM property, we know the following

$$MRS_{w_1, w_2}^h = -\frac{dw_2}{dw_1} = \frac{u'(w - \bar{p}A)(1 - p_h)}{u'(w - \bar{p}A - L + A)p_h},$$

$$MRS_{w_1, w_2}^l = -\frac{dw_2}{dw_1} = \frac{u'(w - \bar{p}A)(1 - p_l)}{u'(w - \bar{p}A - L + A)p_l}.$$

Since we've stipulated that High and Low types are otherwise identical, we know that $u_h(w) = u_l(w)$. This implies that

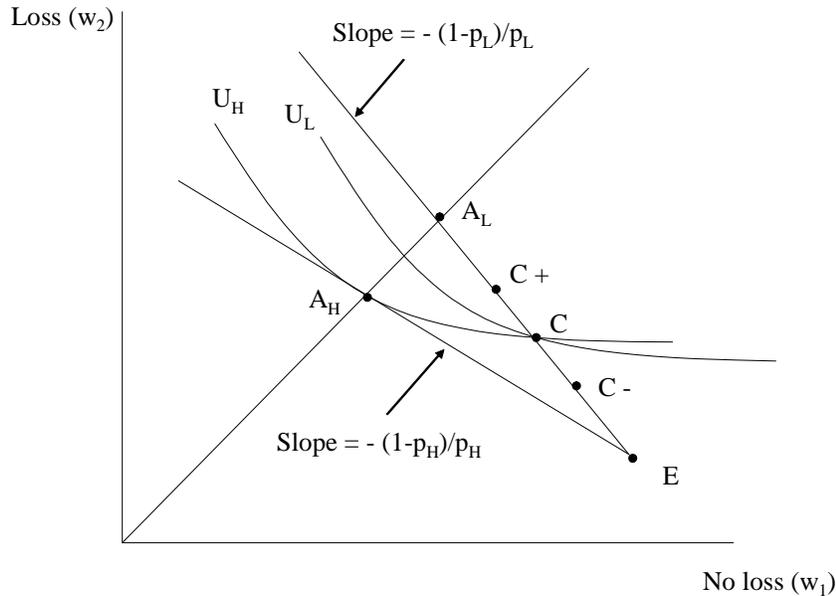
$$\frac{MRS_h}{MRS_l} = \frac{1 - p_h}{p_h} \cdot \frac{p_l}{1 - p_l} = \frac{1 - p_h}{1 - p_l} \cdot \frac{p_l}{p_h} < 1.$$

This shows that the slope of the indifference curve for type h is less steep than for type l .

- Intuitively: Because the probability of loss is lower for type l , type l must receive strictly more income than h in the loss state to compensate for income taken from the no loss state. Even though both h and l types are risk averse, the certainty equivalent of the initial uninsured state is *higher* for l than h types since the l types have lower odds of a loss. This implies that the l types have steeper indifference curves for transfers of income between non-loss and loss states.
- Notice finally that the pooling equilibrium involves a *cross-subsidy* from l to h types (that is, l types pay more than their expected cost and h types pay less than their expected cost). We know there is a cross-subsidy because h, l pay the same premium but h types make more claims. Herein lies the problem...
- Is there anything special about how point A is selected?
 1. It's on the aggregate fair odds line. Otherwise, it is not a break-even policy.
 2. The l indifference curve that intersects A must lie above the l indifference curve that intersects E . If not, l types would prefer no insurance.
 3. The h indifference curve that intersects A must lie above the h indifference curve that intersects E . If not, h types would prefer no insurance. This 3rd condition is automatically met. Because the pooling policy provides a subsidy to h 's and provides some insurance, it must be preferred to E .
- There are many points on the aggregate fair odds line that will meet these criteria and so could be labeled A .

5.1 Failure of the pooling equilibrium

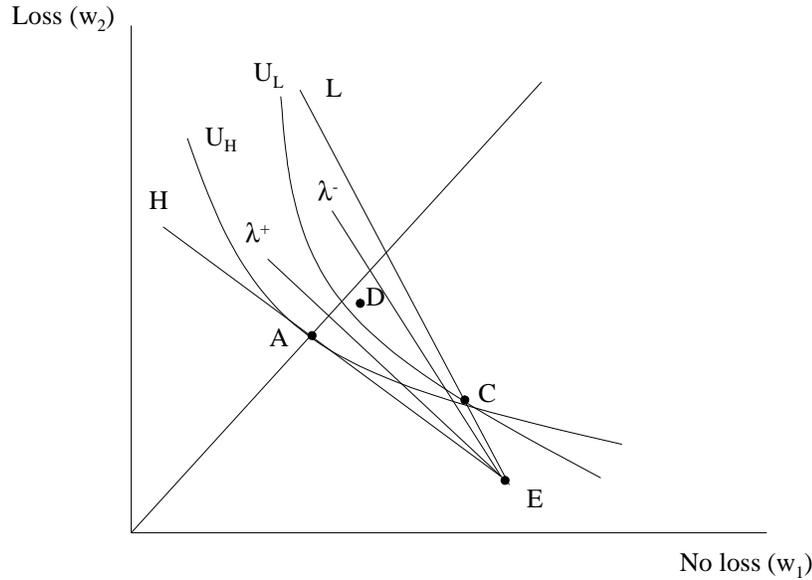
- Although A meets the 1st equilibrium condition (it breaks even), consider the second condition: no potentially competing contract can make a non-negative profit.
- Consider what happens if another insurance company offers a policy like point B in the figure:
- How do h types react to the introduction of B ? They do not. As you can see, B lies strictly below U_h , so clearly h types are happier with the current policy.
- However, l types strictly prefer this policy, as is clear from the fact that B is above U_l . Why is this so? Point B is actuarially a better deal – it lies above the fair odds line for the pooling policy. On the other hand, it doesn't provide as much insurance – it lies closer to E than does point A . This is attractive to l types because they would rather have a little more money and a little less insurance since they are cross-subsidizing the h types. (For the opposite reasons, h types prefer the old policy.)
- So, when policy B is offered, all l types change to B , and the h types stick with A .
- B is profitable if it attracts only l types. That's because it lies below the fair odds line for l types.
- But A cannot be offered without the l types participating – it requires the cross-subsidy.
- Consequently, the pooling equilibrium *does not* exist. It is always undermined by a 'separating' policy that skims off the l types from the pool.
 - Free entry leads to 'cream skimming' of low-risk from the pool.
 - The pooling policy loses money because only high risk types remain.
 - Hence, the pooling policy makes losses and disappears.
- An intuitive explanation for this result: Cross-subsidies are not likely to exist in competitive equilibrium. If a company loses money on one group but makes it back on another, there is a strong incentive to separate the profitable from the unprofitable group and charge them different prices (or just drop the unprofitable group), thereby undermining the cross-subsidy.



6 Candidate separating equilibrium

- Since the pooling equilibrium is infeasible, let's consider instead a 'separating equilibrium.'
- See Figure 3.
- Notice again the two fair odds line corresponding to the two different risk groups.
- Points A_l and A_h are the full-insurance points for the two risk groups. Group l has higher wealth because its odds of experiencing a loss are lower.
- Note the point labeled C on the fair odds line for the l group. This is where the indifference curve from the full-insurance point for the h group crosses the fair odds line for the l group.
- Notice that the indifference curve U_l that intersects point C is steeper than the corresponding curve for the h group. In other words $|MRS_{w_1, w_2}^h| < |MRS_{w_1, w_2}^l|$, as we showed earlier.
- What is special about point C ?
 - Observe that it is the best policy you could offer to the l types that would not also attract h types.

- If a firm offered the policy C^+ on the figure, l types would strictly prefer it – but so would h types, which would put us back in the pooling equilibrium.
- If a firm offered the policy C^- , h types would not select it, but l types would strictly prefer C , the original policy. So, any policy like C^- is dominated by C .
- So, C is the point that defines the ‘separating constraint’ for types h, l . Any policy that is more attractive to h types would result in pooling.
- Notice by the way that any point like C^+ also involves cross-subsidy (if h types take it). We can see this because U_h is the indifference curve for the fully-insured type h . If there is a point that h types prefer to full-insurance (that is actuarially fair of course), it can only mean that the policy is subsidized.
- So we have a candidate equilibrium:
 - Policies A_h and C are offered.
 - Type h chooses A_h and type l chooses C .
 - Both policies break even since each lies on the fair odds line for the insured group.
- Before we ask whether this candidate pair of policies is in fact an equilibrium (according to the criteria above), let’s look at its properties:
 1. High risk types are fully insured.
 2. Low risk only partly insured! (Esp. ironic since they should be ‘easier’ to insure.)
But if a company offered a policy that fully insured the l risk, it would also attract the high risk.
 3. Both types are charged actuarially fair prices. Thus l types pay a lower premium per dollar of coverage (and they also receive lower coverage).
- Preferences of h risk buyers act as a constraint on the market. Firms must maximize the well-being of l buyers subject to the constraint that they don’t attract h buyers.
- Notice also that h risk are no better off for the harm they do to l risk. The externality is entirely *dissipative*, meaning that one group loses but no group gains. This is the opposite of Pareto improvement – and potentially a large social cost.



6.1 Possible failure of the separating equilibrium

- Do the proposed policies constitute a separating *equilibrium*?
- See Figure 4.
- Consider policy D in the figure. Who would buy this policy if offered? *Both* types h and l would buy policy D because it lies above each of their indifference curves when purchasing policies A_l and C .
- What's the potential problem with D ? D is a pooling policy, which we know cannot exist in equilibrium.
- What determines whether D is offered? This depends on whether D is more profitable than the pair A_l, C , both of which offer zero profits. What determines the profitability of D is λ , the share of high risk claimants in the population. To see this, observe that the slope of the fair odds line for pooling policies depends on λ : the larger is λ , the closer the pooling odds line lies to the h fair odds line; the smaller is λ , the closer the pooling odds line lies to the l fair odds line.
- In the above, figure the pooling odds lines corresponding to λ^+ and λ^- (λ^+, λ^- correspond

to two different cases):

- If the population is mostly high risk (λ^+), the pooling policy D that would break the separating equilibrium is unprofitable (lies above the fair odds line) and so will not be offered.
 - By contrast, if the population is mostly low risk (λ^-), the pooling policy D that would break the separating equilibrium is *profitable* (lies below the fair odds line) and so *will be offered*.
- In the latter case, *the model has no equilibrium*.
 - Why does a low value of λ cause the separating equilibrium to fail?
 - At the separating equilibrium, the l risk types are not fully insured, and they are unhappy about this.
 - A pooling policy like D that requires just a little cross-subsidy to h types but offers more insurance is preferred by type l 's to policy C .
 - Hence, if there are sufficiently few h types in the market, a firm could profitably offer this policy and it will dominate the two separating policies.
 - What's ironic here is that it is the l types' risk aversion that makes them willing to tolerate some cross-subsidy to obtain insurance (i.e., they prefer more insurance at an actuarially *unfair* price to less insurance at an actuarially fair price). But the market cannot tolerate cross-subsidy – as we've shown.
 - So, if a policy involving cross-subsidy dominates a set of policies that induces separation, no equilibrium exists.
 - To sum up:
 1. Welfare (efficiency) losses from adverse selection can be high.
 2. The costs appear to be born entirely by the low risk claimants (b/c of the need to get the h risk to select out of the pool). [This is a general result in models with private information: the sellers of low quality goods (in this case, 'high risks') exert a negative externality on the sellers of high-quality goods (low risks). The high quality types typically 'pay' to distinguish themselves from the low-quality sellers. Here, the low-risk types need to be willing to bear above-optimal risk to qualify for the lower cost policy.]

3. Pooling equilibria are unstable/non-existent
4. Separating equilibrium may also not exist.

7 Implications of Rothschild-Stiglitz

- When information is private, the usual efficiency results for market outcomes can be rapidly undermined. The only imperfection in this market is that one set of parties is better informed than another. (Clarify for yourself that there would be no inefficiency if the insurance company could tell who is type l, h). The equilibrium of the RS model violates the first welfare theorem: the free market equilibrium is not Pareto efficient.
- Is this model relevant to real insurance markets? In my view, it is *highly* relevant. The insights of the model are fundamentally correct, though the results are certainly too stark (not a flaw in the model; it's meant to be stylized).
- Health insurance:
 - Health club benefits, maternity benefits—why are these offered?
 - Why do individual policies cost so much more than group policies?
- Auto insurance: You can choose your deductible, but your premium rises nonlinearly as you choose lower and lower deductibles. Why?
- What if MIT allowed new Economics professors to choose between two salary packages: low salary plus guaranteed tenure or high salary but no tenure guarantee. Assume there is no moral hazard problem (i.e., professors don't slack-off if tenured). Will this personnel policy yield a desirable faculty?

8 Why do insurance markets work *at all*? Finkelstein and McGary (2006), “Multiple Dimensions of Private Information: Evidence from the Long-Term Care Insurance Market.”

- Based on the RS paper, you might be left with the thought, “It's a wonder that insurance markets work at all! If even a small degree of adverse selection is enough to destabilize the

market (e.g., there is no equilibrium in RS when almost everybody is a high-risk type save for a few low-risk types), then shouldn't be things much worse in the real world where, presumably, there are many, many dimensions of heterogeneity and private information?

- The Finkelstein-McGary (FM) paper from 2006 provides a highly original set of insights into why insurance markets may function *better* than the RS model would suggest, despite the presence of adverse selection.
- FM does not in any sense overturn RS. It does offer an important insight into the nature of insurance markets, specifically: that *both adverse and advantageous selection* may be present.
 - Adverse selection: Consumers with private information that they are riskier than other observably similar consumers are more likely to buy insurance.
 - Advantageous selection: Consumers who are *less risky* than other observably are more likely to buy insurance (that is, all equal, they are willing to pay more for insurance or are more likely to buy it at a given price).
- Adverse selection tends to arise because 'risky' people can benefit from paying an insurance premium that is lower than their expected cost. This would be true even if these consumers are risk neutral (e.g., if I can buy an auto policy for \$100 a year and my expected accident costs are \$300 per year, that's a policy I'd like to have).
- Advantageous selection may occur if consumers who are more cautious also have greater demand for insurance. This would occur, for example, if cautiousness and risk aversion (the concavity of the utility function over wealth) are positively correlated. This does not seem implausible. People who don't start their cars until they've secured their seat belts and would never drive above the speed limit may also not sleep easily unless they are abundantly insured. People who like to drive with one hand on the wheel and the other on a cold can of beer may not be the worrying type. Thus, it could be the case that 'taste for risk' and 'taste for insurance' are inversely correlated.
- FM explores this idea in the Long Term Care insurance market. (Long Term Care is an extremely expensive proposition. Individuals who require *LTC* can easily use up their entire life savings in a few years time. *LTC* insurance is expensive and not widely purchased in the U.S.)

- The FM analysis proceeds in three interesting and readily interpretable steps:
 1. FM first establish that individuals *do* have informative beliefs about their odds of requiring *LTC*, *and* moreover that these beliefs are *not* captured by the insurance industry’s risk assessment model. Thus, consumers have private information about their riskiness. And moreover, these beliefs predict the purchase of *LTC* insurance; consumers who believe that they are more likely to require *LTC* are more likely to buy a policy. This is direct evidence of adverse selection on private information about risk.
 2. FM next show that, contrary to intuition, consumers’ private information about their *LTC* needs is *unrelated* to the joint probability that they both buy *LTC* insurance and receive *LTC*. That is, consumers who buy *LTC* insurance are not differentially likely to receive *LTC* (*conditional* on the insurance company’s information set — so, private information does not appear important.)
 3. FM resolve this paradox by showing that there is also *advantageous* selection into *LTC* coverage. Individuals who (1) take better care of their health, (2) are wealthier, and/or (3) use their seat belts more often are *less likely* to require *LTC* and *more likely* to buy *LTC* insurance ($I = 1$).
- This paper is easy to understand if we break up the problem into discrete categories. Imagine that there are two types of *LTC* risk populations, *H* and *L*, and two types of risk preference populations, *C* and *R*, for Cautious and Reckless.
- Imagine that a person will buy *LTC* Insurance if she is either *H* or *C*—that is, she is high-risk or cautious.
- The population parameters (describing the fraction of *H* v. *L* and *C* v. *R*) are:

$$\begin{aligned} \Pr(H) &= \lambda_H, & \Pr(L) &= 1 - \lambda_H \\ \Pr(C) &= \delta_c, & \Pr(R) &= 1 - \delta_c \end{aligned}$$

- The probability that a person requires *LTC* are:

$$\begin{aligned} \Pr(LTC|H) &= p_H, \\ \Pr(LTC|L) &= p_L, \\ &\text{with } p_H > p_L. \end{aligned}$$

- Imagine finally that H and L know their types but insurance companies cannot tell them apart. (All that matters here is that applicants have a better guess about H and L than do insurance companies; thus, there is ‘residual’ private information.)
- Now, let’s do the FM analysis in three steps...

8.1 Will H types be more likely to require LTC than L types?

- Yes, this is true by assumption: $p_H > p_L$.

8.2 Will H types be more likely to buy insurance than L types?

- You might be tempted to say yes, but this is not necessarily true. We said that a person buys insurance if either H or C . So, a person buys insurance iff:

$$\begin{aligned}\Pr(I|H) &= 1 \\ \Pr(I|L) &= \Pr(C|L).\end{aligned}$$

So, as long as $\Pr(C|L) < 1$, that is so long as that not all L types are Cautious, then it will be true that H types are more likely than L types to buy insurance. Of course, if $\Pr(C|L) = 1$, then all H and all L types will buy insurance.

- Assuming that $\Pr(C|L) < 1$ would give us a case consistent with FM’s first observation: H types are more likely than L types to buy insurance.
- Again, bear in mind that we are assuming that a consumer’s type, H or L , is known to him or her but unobservable to the insurer. Thus, H or L is residual private information. We could equally well assume that each consumer knows H or L with certainty, and that the insurance company observes H , which is a noisy signal of H , where $\Pr(H = 1|H) = \gamma_H < 1$ and $\Pr(H = 1|L) = \gamma_L > 0$ and $E(H) = \lambda_H$. Here, the consumer’s beliefs about H are still more informative than the insurance company’s assessment of H , so the consumer again has residual private information. We won’t add this extra layer of complexity because it does not add substantively to the model.
- So, we are in a setting where H types have higher probability of buying insurance than L types and are more likely than L types to require LTC .

8.3 Will C types be more likely to buy insurance than R types?

- The answer to this question is analogous to above. A person buys insurance if H or C :

$$\begin{aligned}\Pr(I|C) &= 1 \\ \Pr(I|R) &= \Pr(H|R)\end{aligned}$$

So, as long as *not* all Reckless types are High risk, it will be the case that Cautious people are more likely to buy insurance than Reckless people.

- Thus, we can have both *adverse* selection on H and *advantageous* selection on C operating simultaneously.

8.4 Will insured consumers be more likely than average to require LTC ?

- Here's the subtlety: given both adverse and advantageous selection, it's not clear whether consumers purchasing LTC insurance will be more or less likely than average consumers to require LTC .
- The *baseline* probability that the average consumer requires LTC is equal to $\Pr(LTC) = p_H \lambda_H + p_L (1 - \lambda_H)$.
- The insured are more likely than average to require LTC iff: $\Pr(H|I) > \lambda_H$, in which case $\Pr(LTC|I) > p_H \lambda_H + p_L (1 - \lambda_H)$
- What is $\Pr(H|I)$? This probability is a function of both λ_H and δ_c and their correlation (or, in this discrete case, the probability of their joint occurrence).

8.4.1 Case 1: Adverse and advantageous selection cancel:

- Consider a case where $\frac{1}{2}$ the population is high risk ($\lambda_H = 0.5$) and $\frac{1}{2}$ the population is cautious ($\delta_c = 0.5$). Moreover, assume that 100% of H people are reckless and 100% of L people are cautious. This can be written in the following contingency table:

	Cautious	Reckless	
High Risk	0%	50%	50% = λ_H
Low Risk	50%	0%	50% = $1 - \lambda_H$
	50% = δ_c	50% = $1 - \delta_c$	

What is the probability that a consumer is High risk given that she's insured?

$$\Pr(H|I) = \frac{\Pr(H)}{\Pr(H) + \Pr(C) - \Pr(H = C = 1)}$$

Note that we subtract off the probability that a person is both H and C ($\Pr(H = C = 1)$) because we would otherwise be double-counting these consumers (since they appear in both λ_H and δ_c).

- Plugging in values:

$$\begin{aligned} \Pr(H|I) &= \frac{\lambda_H}{\lambda_H + \delta_c - \Pr(H = C = 1)} \\ &= \frac{0.5}{0.5 + 0.5 + 0} = 0.5 = \lambda_H \end{aligned}$$

- In this case, the advantageous selection by the Cautious offsets the adverse selection by the High risk types. So, the insured do not have higher LTC needs than the baseline consumer.

8.4.2 Case 2: Adverse selection dominates

- Consider a case where $\frac{1}{2}$ the population is high risk ($\lambda_H = 0.5$) and $\frac{1}{2}$ the population is cautious ($\delta_c = 0.5$). Moreover, assume that $\frac{3}{5}$ of H people are reckless and $\frac{3}{5}$ of L people are cautious. So, the contingency table would look as follows:

	Cautious	Reckless	
High Risk	20%	30%	50% = λ_H
Low Risk	30%	20%	50% = $1 - \lambda_H$
	50% = δ_c	50% = $1 - \delta_c$	

Who buys LTC insurance? Everyone who is either High risk or Cautious or both. This is 80% of the population. Consumers who do not buy are neither H nor C (which is 20% of the population).

- What is the probability that a person who has purchased LTC insurance is type H ? This probability is:

$$\Pr(H|I) = \frac{0.5}{0.5 + 0.5 - 0.2} = 0.625 > \lambda_H.$$

- Thus, despite the presence of advantageous selection, it's still the case that those who are insured have higher than baseline probability of LTC .

8.4.3 Case 3: Advantageous selection dominates

- This case cannot occur in our simple setup because all high risk types *always* buy insurance. Thus, the insured population is always *at least as high risk* as the baseline population.
- But we can change model's parameters slightly to consider such a case. Continue to assume as above that 50% of the population is high risk and 50% low risk, and that 40% of high risk are cautious and 60% of low risk are cautious. Now, additionally assume that only 50% of reckless consumers buy insurance. Thus, because high risk consumers are also more likely to be reckless consumers, they are more likely to be the type of consumer who does not buy insurance:

	Cautious	Reckless	
High Risk	20% (100% buy insurance)	30% (50% buy insurance)	50% = λ_H
Low Risk	30% (100% buy insurance)	20% (50% buy insurance)	50% = $1 - \lambda_H$
	50% = δ_c	50% = $1 - \delta_c$	

- What is $\Pr(H|I)$?

$$\Pr(H|I) = \frac{0.2 + 0.3 \times 0.5}{0.2 + 0.3 \times 0.5 + 0.3 + 0.2 \times 0.5} = 0.47 < \lambda_H$$

In other words, the insured have on average lower *LTC* claims than the baseline population.

- Conversely, the non-insured have higher *LTC* claims:

$$\Pr(H|I = 0) = \frac{0.3 \times 0.5}{0.3 \times 0.5 + 0.2 \times 0.5} = 0.60 > \lambda_H.$$

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