Economics 14.04
Problem Set 6 - Due Nov 22nd in class

Notes: This problem set is mostly about screening which is a hard subject. Spend a lot of time thinking about the results you get in problem 1, they will help you think about how to set up and solve problems 2 and 3.

1. Consider the monopolist from the last problem set who is serving two types of consumers with demand functions. Let $c(y) = 0$:

$$
\begin{aligned}
D_L(p,k) &= \begin{matrix} 2-p & k \leq \frac{(2-p)^2}{2} \\ 0 & Otherwise \end{matrix} \\
D_H(p,k) &= \begin{matrix} 4-p & k \leq \frac{(4-p)^2}{2} \\ 0 & Otherwise \end{matrix}
\end{aligned}
$$

Recall from the last problem set that an agents utility from consuming good 1 is given by:

$$
\begin{aligned}
U_L(p,k) &= \frac{(2-p)^2}{2} - k \\
U_H(p,k) &= \frac{(4-p)^2}{2} - k
\end{aligned}
$$

(a) Suppose that the low market all have student IDs that allow them to be differentiated from the high types. For an agent to participate it must be that:

$$
\begin{aligned}
U_L(p,k) &\geq 0 \\
U_H(p,k) &\geq 0
\end{aligned}
$$

These are known collectively as individual rationality (IR) constraints or participation constraints (PC). Draw graphically the following cases. You do not have to do the math for this problem:

i. Suppose legislation requires that $k = 0$ and that $p_L = p_H$. Draw the optimal price and shade the producer surplus.

ii. Suppose legislation requires that $k = 0$ but p_L and p_H can be different. Draw the optimal prices and shade the producer surpluses

iii. Suppose legislation requires $p_L = p_H$ and $k_L = k_H$. Draw the optimal price and shade the producer surpluses. Will the agent serve both markets?

iv. Suppose both p_L, p_H, k_L,and k_H.can all be different Draw the optimal price and shade the producer surpluses. Will the agent serve both markets?

(b) Now suppose that the agent can not tell one type of consumer from the other. In this case we require that each agent type gets more utility from choosing the bundle meant for them than the one meant for the other type of agent:

$$\begin{aligned} U_L(p_L, k_L) &\geq U_L(p_H, k_H) \\ U_H(p_H, k_H) &\geq U_H(p_L, k_L) \end{aligned}$$

These conditions are called incentive compatability constraints. They bascially say that a high type who pretends to be a low type must get less utility than he would acting as a high type. In standard screening problems it is typically the IR constraint of the low type and the IC constraint of the high type that binds. That is:

$$\begin{aligned} U_L(p, k) &\geq 0 \\ U_H(p_H, k_H) &\geq U_H(p_L, k_L) \end{aligned}$$

To think about these concepts draw graphically the following cases. You do not have to do math for this problem. If you haven't already done it, assume that the low market is large enough that it isn't shut out.:

i. Suppose legislation requires $p_L = p_H$ and $k_L = k_H$. Draw the optimal price and shade the producer surpluses. Will the agent serve both markets? Is this different from the problem with full signals above?
ii. Suppose both p_L, p_H, k_L,and k_H.can all be different Draw the optimal price and shade the producer surpluses.
iii. Suppose instead of only being able to restrict p and k the monopolist can also restrict quantity q_L and q_H. Draw the optimal price and shade the producer surpluses.

2. **(From McAfee 1996) In May 1990, IBM announced the introduction of the LaserPrinter E, a lower cost alternative to its popular LaserPrinter. The LaserPrinter E was identical to the original LaserPrinter except that it printed at 5 ppm instead of 10 ppm. According to Jones(1990), the engine and parts of the printer were virtually identical to the faster printer except that the controller for the slower printer had firmwear that inserted wait states to slow the print speed of the printer. This problem is designed to see how such damaging of a product may make everyone better off.

(a) Assume first that you have a single high quality product that you are selling. The product has constant marginal cost to produce of \$1. There are two types of consumers who are willing to buy a single unit of your good. 25% of the population is type 1 and are willing to buy your product if the cost of producing it is less than or equal to \$11. 75% of the population is type 2 and are willing to buy your product if it is priced at \$3.00. Suppose that you must sell your product at a constant price (you can not screen your consumers) - show that you will only sell to the high types.

(b) Now assume that you can produce a broken version of your product. Let s be a measure of how broken your product is. The high types get utility $U_H(x_i, s)$ for consuming one unit of type s good. Thus $U_H(1, 0)$ is the value to the high type of buying one unit of the original quality good and $U_H(1, s)$ would be the utility of the high type for buying a good with quality s. Similarly, the low type get $U_L(1, 0)$ for the original good and $U_L(1, s)$ for consuming a low good of value s. Assume that $\frac{\partial U_H}{\partial s} < \frac{\partial U_L}{\partial s} \leq 0$. Assume that $c(s) \geq 1$ for all $s > 0$ (it is costly to produce an inferior good).

i. Again assume that there is no screening other than offering a low quality good. Set up the monopolists problem.

ii. Suppose that $U_H(1, s) = 11 - 2s$, $U_L(1, s) = 3 - .25s$, and $c(s) = 1$. Find the optimal quality of the lower good and the amount charged for the two goods.

iii. Suppose by law the monopolist can only reduce quality down to $s = 3$. Show that in this case, everyone is at least as well off due to the creation of the damaged good.

3. Suppose that a hospital carries K doses of an opium based medicine used to help patients suffering from micro fibralgia. $N \geq K$ patients come seeking the drug at the same time. 6 of these actually suffer from the disease while N-6 of them are drug addicts looking for a fix.

(a) Suppose that the hospital does not care about the utility of the drug addicts. The utility of a micro fibralgia agent receiving the drug is 10. The utility of a micro fibralgia agent who does not receive the drug is zero. Calculate the total expected utility the hospital can provide to its micro fibralgia patients if it has no way to screen between drug addicts and true patients.

(b) Suppose that the Hospital can force agents to waste time before being treated. The utility of a micro fibralgia user who must wait and receives the medication with probability p is:

$$U_{Micro}(w, p) = 10p - 2w$$

Drug addicts have a utility function of:

$$U_{DA}(w, p) = 4 - w$$

The addicts are not rational - as long as they stay in the hospital they believe that the probability of receiving treatment is 1. Both agents outside option is zero, that is $U_{Micro}(0, 0) = U_{DA}(0, 0) = 0$.

i. Suppose the hospitals objective function is to maximize the utility of its micro fibralgia patients. Argue that the only possible optimal wait time they could choose to impose is w=0,4. (5 points)

ii. Calculate the Expected Utility of the micro fibralgia patients and drug addicts at each of the wait times as a function of N and K. (5 points)

iii. Suppose that K = 10, N=60 - should the hospital screen its patients? Will there be a shortage of medication? (5 points)

4. **Consider a world in which state contingent contracts can be written. There are two people (Chris and Tatiana) and two goods - umbrellas and swim suits. When it is raining, both people like having as many umbrellas as possible. If it is sunny, both people like having as many swim suits as possible. Suppose each person has an identical utility function $U(u, s|raining) = u$, $U(u, s|sunny) = s$ where u and s are the number of umbrellas and swimming suits the agent possesses in the given state (they get more utility from having more umbrellas).

   Assume that at t=0 each agent is endowed with 1 umbrella and 1 swimsuit for the next period. The two agents disagree on the probability of whether it will rain in the future. Tatiana is an optimist and believes it is going to be sunny with probability .75 and rainy with probability .25. Chris is a pessamist and believes it is going to be sunny with probability .5 and rain with probability .5. Both people are vNM utility maximizers, thus:

$$\begin{aligned} U_{Tatiana}(u,s) &= .75U(u,s|sunny) + .25U(u,s|raining), Tatiana'sendowment : u = 1, s = 1 \\ U_{Chris}(u,s) &= .5U(u,s|sunny) + .5U(u,s|raining), Chris'sendowment : u = 1, s = 1 \end{aligned}$$

   (a) Draw the edgeworth box for this problem. What do indifference curves for the two individuals look like?

   (b) Before doing any math - why is it that we expect our final allocation to be on the boundary?

   (c) Why in this problem do we not have a unique price on which all markets clear?

   (d) Suppose that I am a social planner trying to efficiently allocate umbrellas and bathing suits to my housemates. I solve:

$$\max_{u_c,u_t,s_c,s_t} \lambda(U_{Tatiana}(u_t,s_t)) + (1-\lambda)(U_{Chris}(u_c,s_c))$$

$$st\ u_c + u_t = 2, s_c + s_t = 2$$

   Show that all the pareto optimal outcomes are on the boundary of the edgeworth box.

5. Suppose that we are in an economy with two individuals $i = 1, 2$ each with identical utility function $u^i(x_1, x_2) = (x_1 x_2)^{1/2}$.

   (a) Suppose that agent 1 is endowed with $\overline{x_1} = 1, \overline{x_2} = 0$ and agent 2 is endowed with $\overline{x_1} = 0, \overline{x_2} = 2$. At what price ratio $\frac{p_1}{p_2}$ will prices clear in this market? Determine the allocation in this situation.

   (b) Prove that in this economy, the price found in part (a) will clear the market regardless of the initial allocation as long as the total endowment stays the same. What will the contract curve look like?

   (c) Show that the contract curve and the price function are orthoginal in this problem.