Econ 14.04 Fall 2006
Midterm

Answer each of the four questions on a seperate piece of paper. You have 90 minutes to complete the test. Good luck!

1. Uncertainty:

   (a) (10 points) An economic agent's preferences can be represented with the von Neumann-Morgenstern utility function, $u(w+y) = (w+y)^{1/2}$, where w is initial wealth and y is income. Her initial wealth is \$4. She owns a lottery ticket that will be worth \$12 with probability 1/2 and \$0 with probability 1/2.

      1. (3 points) What is her expected utility?
      2. (5 points) What is the lowest price at which she would be willing to sell her lottery ticket?
      3. (2 points) What is the amount of her risk premium for the lottery ticket?

   (b) (15 points) Assume that a risk nuetral agent with vN-M utility function $u(w-L) = w - L$ faces the loss of an amount L with probability π. The economic agent can influence π by exercising caution but this will be costly. Let the cost, C, required to achieve probability $\pi \in [0,1]$ be given by $C = (1-\pi)^2$.

      1. (5 points) If no insurance is available what level of π will the agent choose?
      2. (2 points) An insurance policy that pays the full amount of the loss is now available at a total premium or price ρ (ie, the policy cost is ρ, not ρL). If insurance has already been purchased, what probability of a loss will the agent chose?
      3. (3 points) Given your answer in (ii) what premium must an insurance company charge to break even?
      4. (5 points) Comparing (i) and (iii), prove that as long as the agent is risk neutral, she will never purchase insurance at the premium which must be charged for the company to break even. What feature of actual insurance policies does this result help explain?

2. Utility maximzation:

   (a) (25 points) Consider the following utility function:

   $$e^{(2x^{1/2}+y)}$$

   Subject to:

   $$p_x x + p_y y = m$$

      1. (5 points) What property of utility makes this problem easier? Which of the inequality constraints will bind?
      2. (15 points) Determine the marshallian demand functions and the indirect utility function (give me the indirect utility of the original problem, not the transformed one). If you get stuck on the math be sure to tell me the steps you would use to finish the problem.

3. (3 points) Assume that there are five agents in the economy have endowments $m^i, i \in \{1,2,3,4,5\}$. Suppose that each agent has $m > \frac{p_y^2}{p_x}$. Find the economy demand for both goods (ie $X(p_x, p_y, m^i) = \sum x^i(p_x, p_y, m^i)$). What form of the utility function simplifies this problem?
4. (2 points) Suppose agents m^1 and m^2 have income $m^i < \frac{p_y^2}{p_x}$. Determine $X(p_x, p_y, m^i)$ and $Y(p_x, p_y, m^i)$

(b) (20 points) Suppose that you are given the following indirect utility function:

$$v(p_x, p_y, m) = \begin{cases} -\infty & m < 2p_x \\ \left(\frac{m-2p_x}{2p_x}\right)\left(\frac{m-2p_x}{2p_y}\right) & m \geq 2p_x \end{cases}$$

1. (5 points) Find the expenditure function for this function
2. (5 points) Find hicksian demand functions
3. (5 points) Find the marshallian demand functions
4. (5 points) Find a utility function that could generate this indirect utility function

(c) (10 points) Assume that an agent has strictly convex and weakly monotonic preferences. We observe that when prices are $p_x = p_y = 10$, the agent chooses to consume $x = 10, y = 10$. Assume that the agents preferences satisfy WARP. State whether bundle 1 is $\precsim, \succsim$, or uncomparable to bundle 2

1. (3 points) Bundle 1: $(5,5)$, Bundle 2: $(6,6)$
2. (3 points) Bundle 1: $(8,11)$, Bundle 2: $(4,14)$
3. (4 points) Bundle 1: $(6,12)$, Bundle 2: $(10,11)$

3. Production

(a) (20 points) Suppose that a firm has access to two production technologies for the production of some good x:

$$\begin{aligned} c_1(x_1) &= \frac{1}{2}\omega_1 x_1^2 \\ c_2(x_2) &= \omega_2 x_2 \end{aligned}$$

1. (10 points) Given a required output $X = x_1 + x_2$ and the fact that $x_1 \geq 0, x_2 \geq 0$, find the amount of $x_1(\omega_1, \omega_2, X)$ and $x_2(\omega_1, \omega_2, X)$ that will minimize cost for any prices and required output level X. Denote the function $c(\omega_1, \omega_2, X)$ as the minimum cost conditional of producing X units of a good.
2. (5 points) Suppose that $\omega_1 = \omega_2 = 1$. the firm uses the good produced in part (i) to produce a good y according to the following function:

$$y = px^{1/2}$$

Calculate the profit maximizing output $y(p)$ and $\pi(p)$.

3. (5 points) Suppose that $p = 2, \omega_1 = \omega_2 = 1$. The workers at plant 1 suddenly go on strike. Management is able to hire a small amount of picket crossers to work the machines but are constrained in their capacity for good 1 at $x_1 \leq .5$. How much would the manager of the plant be willing to pay to have the plant produce .5 more units of output? How much would the manager of the plant be willing to pay to have the plant produce .25 more units of output?
4. (Bonus 5 points) Suppose that $p = 2, \omega_1 = \omega_2 = 1$. Come up with a function $W(x_1)$ that gives the willingness to pay to produce x_1 units from production technology 1.