MIT 14.04 Intermediate Microeconomie Theory
Fall 2005

Exam #1

Instructions

Answer each of the four problems on a seperate piece of paper. You will have 1.5 hours to complete the exam.

Problem 1 (30 minutes)

Consider a consumer with a utility function $u(x_1, x_2) = e^{(x_1 + \ln(x_2))^{1/3}}$

(3 points) a) What properties about utility functions will make this problem easier to solve?

(3 points) b) Which of the non negativity input demand constraints will bind for small m?

(10 points) c) Derive for the marshallian (uncompensated) demand functions and the indirect utility function.

(3 points) d) Derive the expenditure function in terms of original utils u.

(6 points) e) Suppose that there are 5 people in the economy each with endowments m^i, $i = 1,2,3,4,5$.

i) Suppose that $m^i > p_1\ \forall i$. Construct the aggregate demand function for x_1 and x_2. '
What properties do the individual demand functions have that simplifty this problem?

ii) Now suppose that $m^1 < p_1$, $m^2 < p_1$, $m^i > p_1$ for $i = 3,4,5$.
Construct the aggregate demand for x_1, x_2

Problem 2 (25 minutes)

Consider a firm with a production function of $f(x_1, x_2) = \min(2x_1, x_1 + x_2)$

(5 points) a) Show that this function is homogeneous of degree 1.

i) What does this imply about the structure of the cost function?

ii) What does this imply about the returns to scale of the technology?

(10 points) b) Derive the cost function for the firm. What are the conditional factor demands?

(5 points) c) Suppose that $p = 2.5, w_1 = 3, w_2 = 1$. If we introduce the constraint that $x_2 < K$.
How much of the good will a profit maximizing producer produce?

Problem 3 (15 minutes)

(5 points) a) In producer theory, suppose we have the conditional factor demand $x_1(w,y) = ln(\frac{w_2}{w_1})y$ for $w_2 \geq w_1$. Assume that when $w_1 = w_2$, $x_2 = y$.
Construct $x_2(w,y)$ when $w_2 \geq w_1$

(5 points) b) In consumer theory, suppose that that a consumer with utility $U(x_1, x_2)$ is given an initial endowment $\overline{x_1}, \overline{x_2}$ so that his total budget is $p_1\overline{x_1} + p_2\overline{x_2}$.

i) Write down or solve for the slutsky equation with endowments.

ii) Suppose that p_1, p_2 are such that consumption is exactly equal to the initial endowment.
Show that $\frac{\partial x_1}{\partial p_2} = \frac{\partial x_2}{\partial p_1}$ in this specialized case.

Problem 4 (15 minutes)

Assume that there is a consumer with weakly monotonic, convex preferences and who is a utility maximizer.

For each of the following pairs of bundles, specify if bundle 1 is $\succsim, \precsim$, or uncomparable to bundle 2.

(2 points) a) Suppose you have no data:

i) Bundle 1: $x_1 = 3, x_2 = 3$, Bundle 2: $x_1 = 6, x_2 = 2.5$

ii) Bundle 1: $x_1 = 3, x_2 = 3$, Bundle 2: $x_1 = 2.5, x_2 = 2.5$

(4 points) b) Suppose that you observe that when $p_1 = 1, p_2 = 1, m = 10$ the consumer chooses $x_1 = 2, x_2 = 8$

i) Bundle 1: $x_1 = 4$, $x_2 = 1$, Bundle 2: $x_1 = 3$, $x_2 = 6$

ii) Bundle 1 $x_1 = 6, x_2 = 4$, Bundle 2: $x_1 = 3$, $x_2 = 8$

(4 points) c) Suppose that we have two observations. When $p_1 = 1, p_2 = 1, m = 10$ the consumer chooses $x_1 = 2, x_2 = 8$. When $p_1 = 1, p_2 = 3, m = 15$ the consumer chooses $x_1 = 15, x_2 = 0$

i) Bundle 1: $x_1 = 5, x_2 = 2$, Bundle 2: $x_1 =.0, x_2 = 2.5$

ii) Bundle 1: $x_1 = 5, x_2 = 2$, Bundle 2: $x_1 =.6.5, x_2 = 0$