

14.05 Lecture Notes
Crises and Multiple Equilibria

George-Marios Angeletos

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1 Obstfeld (1996): self-fulfilling currency crises

What triggers speculative currency crises? Obstfeld emphasizes the coordination problem that emerges because of lack of commitment on the side of the government and the presence of multiple small speculators.

1.1 Basic Barro-Gordon game

A large number of private agents play against a government.

Government. The government solves

$$\min L = \{(y - y^*)^2 + \beta\pi^2\}$$

subject to the Philips curve

$$y = \bar{y} + \alpha(\pi - \pi_e).$$

Private agents. The agents set π_e to minimize $\mathbb{E}(\pi_e - \pi)^2$

Information and timing. The agents move first, setting π_e . The government moves last, setting π after observing π_e .

Best responses. The best response of the government is given by

$$\pi = g(\pi_e) = \frac{\alpha(y^* - \bar{y}) + \alpha^2\pi_e}{\alpha^2 + \beta}.$$

The best response for the agents is

$$\pi_e = \mathbb{E}\pi$$

Equilibrium. In equilibrium,

$$\pi_e = \mathbb{E}g(\pi_e) = \frac{\alpha(y^* - \bar{y}) + \alpha^2\pi_e}{\alpha^2 + \beta} \equiv G(\pi_e).$$

By the properties of g , we have $G(0) > 0$ and $G' \in (0, 1)$. Hence, there is a unique fixed point with $\pi_e > 0$. Indeed, this is given by

$$\pi_e = \frac{\alpha(y^* - \bar{y})}{\beta}.$$

It follows that equilibrium inflation and output are given by

$$\pi = \pi_e \quad \text{and} \quad y = \bar{y}.$$

1.2 Adapting Barro-Gordon to exchange-rate policy

Obstfeld (1996) reinterprets π as the rate of exchange-rate depreciation and adds a fixed cost to abandoning the peg.

- Maintaining the currency peg means $\pi = 0$; abandoning means $\pi \neq 0$.
- We modify the preferences of the government so that

$$L = \{(y - y^*)^2 + \beta\pi^2 + \theta D\}$$

- D is an indicator that takes the value 1 if $\pi \neq 0$ (devaluation) and 0 if $\pi = 0$.
- θ represents the value of maintaining the peg.
- We can think of θ as value of “reputation”. However, this raises the question of how one should model reputation from first principles. E.g., in repeated games, or games of incomplete info. We abstract from this issue.
- The timing is the same: first, agents set π_e ; next, the government chooses π .

Optimal exchange-rate policy. The government now faces a discrete choice: abandon or not abandon the peg. (this is an example of "regime switch").

- If the government abandons the peg, then it's optimal to set $\pi = g(\pi_e) > 0$, where g is defined as in the previous section. Her losses are then given by

$$L = L_{flex}(\pi_e, \theta) \equiv \frac{\beta}{\alpha^2 + \beta} (y^* - \bar{y} + \alpha\pi_e)^2 + \theta.$$

- If instead the government maintains the peg ($\pi = 0$), then her losses are given by

$$L = L_{fixed}(\pi_e) \equiv (y^* - \bar{y} + \alpha\pi_e)^2$$

- It follows that the government abandons the peg if and only if

$$L_{flex}(\pi_e, \theta) \geq L_{fixed}(\pi_e),$$

or equivalently

$$\theta \leq \frac{\alpha^2}{\alpha^2 + \beta} (y^* - \bar{y} + \alpha\pi_e)^2.$$

- Note that a higher π_e makes it more likely that the government will abandon the peg.
- But whenever the government is expected to abandon the peg, private agents expect high inflation and hence find it optimal to set a high π_e
- Hence, as long as θ is not too large, there can be **multiple equilibria!**
- one in which $\pi_e = 0$ and the government maintains the peg
- and another in which $\pi_e = \pi_e^*$ and the government abandons the peg.

2 Calvo (1988): debt repudiation (or debt deflation)

A Barro-Gordon-like model, with the option to default on *domestic* debt.

Stage 0: Government borrows $b = \bar{b}$ (exogenous) at rate R_b (to be determined in equilibrium)

Stage 1: Government decides fraction ϕ of $R_b b$ to be repudiated.

- Ex post budget:

$$T = g + (1 - \phi(1 - \theta))R_b b$$

T = distortionary taxes (endogenous)

g = government spending (exogenous)

θ = cost of repudiation per unit of debt repudiated (here exogenous, but... think about it)

- “Welfare” is given by the consumption of the representative agent:

$$c = y - \Lambda(T) + Rk + (1 - \phi)R_b b - T$$

$y - \Lambda(T)$ = income net of cost of distortionary taxes

R = exogenous rate of return on capital (outside opportunity, pins down risk-free rate)

Equilibrium. In stage 1, the government choose $\phi \in [0, 1]$ so as to maximize c , with R_b given from stage 0. Since debt is domestic, repudiation of debt is a transfer within the economy. But it can be desirable from the perspective of a benevolent government because it allows the government to lower distortionary taxation.

- For $\theta \in (0, 1)$, there is a range $[\underline{R}_b, \bar{R}_b]$ such that:
 - whenever $R_b < \underline{R}_b$, optimal to set $\phi = 0$;
 - whenever $R_b \in (\underline{R}_b, \bar{R}_b)$, optimal $\phi \in (0, 1)$ and increasing in R_b ;
 - whenever $R_b > \bar{R}_b$, optimal to set $\phi = 1$.

That is, the incentive to repudiate, and hence the optimal ϕ , increases with R_b .

- But... since agents are indifferent between k and b , equilibrium R_b must satisfy

$$(1 - \phi)R_b = R$$

It follows that R_b increases with (expected) ϕ .

- Hence... **multiple equilibria!** (for intermediate values of θ)

Remark. Here repudiation was explicit; but it works similarly if it is implicit in the form of deflating nominal debt. See Section II of Calvo (1988).

3 Diamond-Dybvig (1983): bank runs

We now consider a variant of Diamond-Dybvig's models of bank runs. This variant abstracts from the design of optimal deposit contracts and focuses on the coordination problem.

Setting. There are three periods, $t \in 0, 1, 2$; a bank that accepts demand deposits and invests them either in a liquid low-return asset or an illiquid high-return asset; and a mass 1 of agents, who each deposit 1 unit of wealth at $t = 0$ and then decides whether to withdraw at $t = 1$ or $t = 2$.

Agents can be of two types: "impatient" agents die at $t = 1$ and hence must consume at $t = 1$, while "patient" agents die at $t = 2$ and are indifferent about the timing of their consumption.

An agent's type is realized in the begging of $t = 1$ and is private info to him. Let $\lambda \in (0, 1)$ be the fraction of patient agents.

Let L be the liquid position of the bank, I the illiquid position, and R the rate of return on the illiquid assets. Also, suppose that the liquidation value of illiquid assets is $\delta < 1$.

(The variables λ, L, I, R, δ are all exogenous.)

Payoffs. Let K denote the mass of agents withdrawing at $t = 1$, $\pi_1(K)$ the payoff for an agent (patient or impatient) who withdraws at $t = 1$, and $\pi_2(K)$ the payoff for a *patient* agent who withdraws at $t = 2$ (the payoff for an *impatient* agent who withdraws at $t = 2$ is 0).

Payoffs depend on K as follows:

$$K < L \quad \Rightarrow \quad \pi_1(K) = 1, \quad \pi_2(K) = \frac{RI + (L - K)}{1 - K}$$

(Any remaining liquidity for $t = 1$ is invested at zero interest and then at $t = 2$ it is distributed together with RI to the agents who waited.)

$$L < K < L + \delta I \quad \Rightarrow \quad \pi_1(K) = 1, \quad \pi_2(K) = \frac{RI - R\delta^{-1}(K - L)}{1 - K}$$

(To pay for the agents withdrawing at $t = 1$, $\delta^{-1}(K - L) < I$ units of the illiquid asset are liquidated.)

$$K > L + \delta I \quad \Rightarrow \quad \pi_1(K) = \frac{L + \delta I}{K}, \quad \pi_2(K) = 0.$$

(Even after liquidating all illiquid assets, there is not enough to meet all withdrawals.)

Equilibrium. Clearly, impatient agents are non-strategic: it is always dominant for them to withdraw at $t = 1$. Hence, in what follows we focus on the incentives of the patient agents.

Let A denote fraction of patient agents who "run" (i.e., withdraw at $t = 1$); the total mass of agents withdrawing at $t = 1$ is

$$K = \lambda + (1 - \lambda)A,$$

The **net payoff from running** can then be expressed as:

$$u(A) = \pi_1(\lambda + (1 - \lambda)A) - \pi_2(\lambda + (1 - \lambda)A)$$

Everybody running is an equilibrium iff $u[1] \geq 0$.

Nobody running is an equilibrium iff $u[0] \leq 0$.

When is everybody running an equilibrium?

Note that $L + \delta I > 1 \Rightarrow u[1] = 1 - \infty < 0$, while $L + \delta I < 1 \Rightarrow u[1] = (L + \delta I) - 0 > 0$.

It follows that everybody running is an equilibrium iff $L + \delta I < 1$.

When is nobody running an equilibrium?

$$\begin{aligned} \lambda < L &\Rightarrow u[0] = 1 - \frac{RI + L - \lambda}{1 - \lambda} < 0 \\ L < \lambda < L + \delta I &\Rightarrow u[0] = 1 - \frac{RI - R\delta^{-1}(\lambda - L)}{1 - \lambda} \equiv g(\lambda) \\ \lambda > L + \delta I &\Rightarrow u[0] = \frac{L + \delta I}{1 - \lambda} - 0 > 0 \end{aligned}$$

Let $\bar{\lambda}$ be the solution to $g(\lambda) = 0$; that is, $\bar{\lambda} \equiv \frac{R\delta^{-1}(L+\delta I)-1}{R\delta^{-1}-1}$. Note that $g(\lambda) < 0$ iff $\lambda < \bar{\lambda}$. Also note that $L + \delta I < 1$ implies $\bar{\lambda} < L + \delta I < 1$, while $L + \delta I > 1$ implies $\bar{\lambda} > L + \delta I > 1$.

If $L + \delta I > 1$, then for all λ we have $\lambda < L + \delta I$ and $g[0] < 0$, which ensures that $u[0] < 0$; hence nobody running is an equilibrium.

If instead $L + \delta I < 1$, then $\bar{\lambda} \in (0, 1)$ and $u[0] < 0$ for $\lambda < \max\{L, \bar{\lambda}\}$ but $u[0] > 0$ for $\lambda > \max\{L, \bar{\lambda}\}$; hence nobody running is an equilibrium for $\lambda < \max\{L, \bar{\lambda}\}$ but not for $\lambda > \max\{L, \bar{\lambda}\}$.

Multiplicity. Whenever $L + \delta I < 1$ and $\lambda < \bar{\lambda}$, both nobody running and everybody running are equilibria.

Three regions of fundamentals. Fix $\lambda \in (0, 1)$, $R > 1$, and $\delta < 1$, and, to simplify, assume $\delta I < \lambda$.

Next, let $\bar{L} \equiv 1 - \delta I$ and \underline{L} be the solution to $\lambda = \max\{L, \bar{\lambda}(L)\}$, and note that, under the above assumption, $0 < \underline{L} < \bar{L} < 1$.

- Whenever $L < \underline{L}$, everybody running is the unique equilibrium.
- Whenever $L \in (\underline{L}, \bar{L})$, there are multiple equilibria.
- Whenever $L > \bar{L}$, nobody running is the unique equilibrium.

We can thus identify the interval (\underline{L}, \bar{L}) as the critical region of fundamentals for which the banking system is solvent (from a fundamentals-perspective) but could be vulnerable to a self-fulfilling liquidity crisis.

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