# 14.05 Review

- Endogenous growth
- Ricaridan equivalence and optimal taxation
- Social insurance
- Business cycles: productivity shocks
- Unemployment
- Money: neoclassical neutrality
- Money: short run real effects

# Endogenous growth

- Solow model and Ramsey model:
  - Conditional convergence to steady state in the long-run
  - growth in GDP per capita: technological progress
- Endogenous growth
  - AK model
  - Learning by doing
  - R&D

# AK model

- Setting as in Ramsey model

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$st: \quad c_t + k_{t+1} \leq f(k_t) + (1-\delta)k_t$$

- Main difference

$$f(k_t) = Ak_t$$

$$r_t = f'(k_t) = A$$

(violates Inada condition)

# AK model: solution

- Euler equation:

$$\frac{c_{t+1}}{c_t} = [\beta(1 + A - \delta)]^{\theta}$$

- Guess linear policy functions: for some $s \in (0, 1)$

$$c_t = (1 - s)(1 + A - \delta)k_t$$

$$k_{t+1} = s(1 + A - \delta)k_t$$

- which implies

$$\frac{c_{t+1}}{c_t} = \frac{k_{t+1}}{k_t} = \frac{y_{t+1}}{y_t} = [\beta(1 + A - \delta)]^{\theta}$$

# AK: solution II

- Resource constraint

$$\frac{c_t}{k_t} + \frac{k_{t+1}}{k_t} = (1 + A - \delta)$$

$$\frac{c_t}{k_t} = (1 + A - \delta) - [\beta (1 + A - \delta)]^{\theta} = (1 - s)(1 + A - \delta)$$

$$\implies s = \beta^{\theta}(1 + A - \delta)^{\theta - 1} = \beta^{\theta}(1 + R)^{\theta - 1}$$

- income and substitution effects: $\theta \lesseqgtr 1$.
- Parameters affect growth rate (as opposed to Solow or Ramsey)

$$\frac{c_{t+1}}{c_t} = \frac{k_{t+1}}{k_t} = \frac{y_{t+1}}{y_t} = [\beta (1 + A - \delta)]^{\theta}$$

increasing in $A$, $\theta$ and $\beta$.

# Learning by doing

- Main difference: externalities.
- Output for firm $m$:

$$Y_t^m = F(K_t^m, h_t L_t^m)$$

- where

$$h_t = \eta \frac{K_t}{L_t}$$

- Important: decentralied market equilibrium difference from social planner.

# Market

- Each firm takes $h_t$ as given, so optimization yields

$$r_t = F_1'(K_t^m, h_t L_t^m)$$

$$w_t = F_2'(K_t^m, h_t L_t^m) h_t$$

- then plug in $h_t = \eta k_t$

$$r_t = f'(\eta^{-1}) = A$$

- Euler equation

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta(1 + r_t - \delta) = \beta(1 + A - \delta)$$

- growth

$$\frac{c_{t+1}}{c_t} = \frac{k_{t+1}}{k_t} = \frac{y_{t+1}}{y_t} = g_t^{CE} = [\beta(1 + A - \delta)]^{\theta}$$

# Social Planner

- Internalizes effect of capital accumultion on $h_t$. First plug in $h_t = \eta k_t$

$$y_t = \frac{Y_t}{L_t} = F(k_t, h_t) = f\left(\frac{k_t}{h_t}\right) h_t = f(\eta^{-1})\eta k_t = A^* k_t$$

$$A^* = f(\eta^{-1})\eta > A \quad \text{(why?)}$$

- So Euler condition

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta(1 + A^* - \delta)$$

- and growth

$$\frac{c_{t+1}}{c_t} = \frac{k_{t+1}}{k_t} = \frac{y_{t+1}}{y_t} = g_t^{SP} = [\beta(1 + A^* - \delta)]^\theta > g_t^{CE}$$

# R&D

- Main idea: technological progress as an economic activity

$$\max_{z_t} q(z_t) V_{t+1} - w_t z$$

$$\Longrightarrow z_t = g\left(\frac{V_{t+1}}{w_t}\right)$$

- Value of innovation

$$V_{t+1} = \gamma \hat{\nu} A_t$$

- Cost of innovation

$$w_t = A_t$$

# R&D II

- Aggregate rate of innovation

$$\lambda_t = q\left(g\left(\gamma\hat{\nu}\right)\right)$$

- Aggregate growth

$$\frac{A_{t+1}}{A_t} = 1 + \gamma\lambda = 1 + \gamma q\left(g\left(\gamma\hat{\nu}\right)\right)$$

- If we increase $\hat{\nu}$ innovation becomes more attractive. If we increase $\gamma$ innovation becomes more attractive and, in addition, it has larger aggregate effects.
- Optimal patent protection: incentives vs. externalities.

# Ricardian Equivalence

- Main idea: timing of taxes has no effects on equilibrium.
- Take government expenditures as given.
- Assume taxes are not distortionary.
- No financial fricitons: households can borrow and lend freely.

# Ricardian equivalence

- Household intertemporal budget constraint

$$\sum_{t=0}^{T} q_t c_t \leq (1 + R_0)a_0 + \sum_{t=0}^{T} q_t w_t l_t - \sum_{t=0}^{T} q_t T_t$$

- and assets $a_t = b_t + k_t$.
- Government budget constraint

$$\sum_{t=0}^{T} q_t g_t + (1 + R_0)b_0 = \sum_{t=0}^{T} q_t T_t$$

- therefore any tax plan that satisfies the government budget constraint leaves the household budget constraint unchanged

$$\sum_{t=0}^{T} q_t c_t \leq (1 + R_0)k_t + \sum_{t=0}^{T} q_t w_t l_t - \sum_{t=0}^{T} q_t g_t$$

- and so households don't change their consumption/ work/ savings decisions.

# Optimal taxation

- Ricardian equivalence fails if there are financial frictions (pset)
- or if taxation is distortionary (also see pset)

$$y_t = Y - \Lambda(T_t)$$

- To simplify assume linear preferences $u(c) = c$, so that $(1 + R_t)\beta = 1$, or $q_t = \beta^t$.
- So now the problem of optimal taxation is to

$$\max_{\{T_t\}} \sum_{t=0}^{T} \beta^t \left(Y - \Lambda(T_t) - g_t\right)$$

$$st: \quad \sum_{t=0}^{T} \beta^t g_t = \sum_{t=0}^{T} \beta^t T_t$$

# Taxation smoothing

- We get taxation smoothing

$$\Lambda'(T_t) = \lambda \Longrightarrow T_t = T^* \; \forall t = 0, 1 \ldots T$$

$$\Longrightarrow T^* = (1 - \beta) \sum_{t=0}^{T} \beta^t g_t$$

- Permenent increase in $g$
- Transitory increase in $g$

# Social insurance

- Main idea: taxation and redistribution can provide ex-ante insurance (before we know whether we will be lucky/ succesful).

$$u_i = -\exp\left\{-\left(c_i - \frac{n_i^{1+\epsilon}}{1+\epsilon}\right)\right\}$$

$$c_i = (1-\tau)y_i + T = (1-\tau)\left(n_i + \nu_i\right) + T$$

- So FOC

$$n_i = (1-\tau)^{\frac{1}{\epsilon}}$$

- to simplify $\epsilon = 1$ so

$$n_i = (1-\tau)$$

# Social insurance II

- Government budget

$$T = \tau \int y_i di = \tau(1-\tau)$$

- Then agent's utility

$$u_i = -\exp\left\{-\left(\frac{1}{2}(1-\tau)^2 + (1-\tau)\nu_i + \tau(1-\tau)\right)\right\}$$

- Maximize its expectation (before knowing $\nu_i$)

$$\max \mathbb{E}\left[u_i\right] = -\exp\left\{-\left(\frac{1}{2}(1-\tau)^2 - \frac{1}{2}(1-\tau)^2\sigma^2 + \tau(1-\tau)\right)\right\}$$

- Optimal $\tau$ increases with $\sigma$ (agents don't like risk).

# Business Cycles: productivity shock

- Use graph of labor market and market for capital services: $\frac{w}{P}$ and $\frac{R}{P}$
- Increase in productivity:
  - higher real wage $\frac{w}{P}$, and hours worked $L$.
  - higher rental price $\frac{R}{P}$ and capital utilization $\kappa K$.
  - Consumption: income vs substitution
    - Permanent shock $C$ and $I$ both go up, so $K$ goes up as well.
    - transitory shock: $C$ could go up or down, $I$ goes up, and hence $K$ as well

# Search model of unemployment

- Job finding (50% of $U$): workers receive wage offers $\frac{w}{P}$, and have a reservation wage $\omega$.
  - from unemployment income
  - from option value of waiting for a better offer
  - Productivity shock improves offers more than the reservation wage: more job finding.
- Job-separation (3% of $L$)
- Natural unemployment rate $u = \frac{U}{U+L}$

$$\phi U = \sigma L = \sigma(U + L - U)$$

$$(\phi + \sigma)U = \sigma(U + L)$$

$$u = \frac{\sigma}{\phi + \sigma}$$

- Job vacancies procyclical.

# Money neutrality in neoclassical model

- Dichotomy: real variables independent of nominal

$$\frac{M}{P} = L(Y, i)$$

$$i = r + \pi$$

- Money neutrality: permanent change in $M \implies$ permanent proportional change in $P$.
- Constant growth rate $\mu$ for $M$ leads to constant inflation rate $\pi = \mu$

$$\frac{M}{P} = L(Y, r + \pi)$$

- So changes in $\mu$ affect $P$ right away.

# Misperception model

- Money has real effects in the short-run
  - misperception model (here)
  - new-keynesian model
- average price level is $P$, but workers think it's $P^e$, so they supply labor according to

$$\frac{W}{P^e} = \frac{W}{P}\left(\frac{P}{P^e}\right)$$

- Long-run: $P^e = P$.
- Short-run: $P^e$ fixed... then it adjusts towards $P$.

# $M$ has real effects in the short run

- Increase in $M$ to $(1+\Delta)\, M_0$ should increase prices to $(1+\Delta)\, P_0$
- but since $P^e = P_0$ is fixed, employment $L$ and hence output $Y = F(K, L)$ go up.
- So prices don't go up by as much, in the short run $P_0 < P_{SR} < (1+\Delta)\, P_0$
- Eventually workers adjust their $P^e = P$, employment and output fall back to their long-run level, and $P \to (1+\Delta)\, P_0$
- money still neutral in the long-run
- only unexpected changes in $M$ have real effects

# Important papers

- Acemoglu et al.: The colonical origins of comparative economic development
  - Main idea: Institutions are really important
  - To show this, look at institutions built by European powers in different colonies
  - where settler mortality was low, they established lots of Europeans and good institutions
  - where settler mortality was high, they couldn't, so they set up (bad) extractive institutions.
- Angeletos and Alessina:
  - if people think income is luck, they tax, so work doesn't pay off and income is mostly luck.
  - if instead they think its effort, they don't tax, so work pays off and it's mostly effort.
  - multiple equilibria: Europe vs USA