

# Final Exam Answers

Course 14.06 – Intermediate Applied Macroeconomics

## 1. Question #1

(a) The firms problem is as follows:

$$\max_{K,L} \Pi = A_t K_t^\alpha L_t^{1-\alpha} - r_t K_t - w_t L_t$$

The FOCS can be written as:

$$\begin{aligned} r_t &= \alpha A_t k_t^{\alpha-1} \\ w_t &= (1-\alpha) A_t k_t^\alpha \end{aligned}$$

Plugging for  $A_t$ , we have:

$$\begin{aligned} r_t &= \alpha g_t^{1-\alpha} k_t^{\alpha-1} \\ w_t &= (1-\alpha) g_t^{1-\alpha} k_t^\alpha \end{aligned}$$

Now, we plug these into  $g_t = \tau^k r_t k_t + \tau^w w_t$ , and we have:

$$\begin{aligned} g_t &= \tau^k \alpha g_t^{1-\alpha} k_t^\alpha + \tau^w (1-\alpha) g_t^{1-\alpha} k_t^\alpha \\ g_t &= \left[ \alpha \tau^k + (1-\alpha) \tau^w \right]^{\frac{1}{\alpha}} k_t \end{aligned}$$

We can now plug this back into  $y_t = g_t^{1-\alpha} k_t^\alpha$

$$y_t = \left[ \alpha \tau^k + (1-\alpha) \tau^w \right]^{\frac{1-\alpha}{\alpha}} k_t$$

We can also plug  $g_t$  into our equations for  $r_t$  and  $w_t$  to get:

$$\begin{aligned} r_t &= \alpha \left[ \alpha \tau^k + (1-\alpha) \tau^w \right]^{\frac{1-\alpha}{\alpha}} k_t^{\alpha-1} \\ w_t &= (1-\alpha) \left[ \alpha \tau^k + (1-\alpha) \tau^w \right]^{\frac{1-\alpha}{\alpha}} k_t^\alpha \end{aligned}$$

And, the resource constraint can be written as:

$$k_{t+1} = y_t - c_t - g_t$$

$$k_{t+1} + c_t = \left( \left[ \alpha \tau^k + (1-\alpha) \tau^w \right]^{\frac{1-\alpha}{\alpha}} - \left[ \alpha \tau^k + (1-\alpha) \tau^w \right]^{\frac{1}{\alpha}} \right) k_t$$

So, in sum, your answer should be:

$$r_t = \alpha \left[ \alpha \tau^k + (1-\alpha) \tau^w \right]^{\frac{1-\alpha}{\alpha}}$$

$$w_t = (1-\alpha) \left[ \alpha \tau^k + (1-\alpha) \tau^w \right]^{\frac{1-\alpha}{\alpha}} k_t$$

$$g_t = \left[ \alpha \tau^k + (1-\alpha) \tau^w \right]^{\frac{1}{\alpha}} k_t$$

$$y_t = \left[ \alpha \tau^k + (1-\alpha) \tau^w \right]^{\frac{1-\alpha}{\alpha}} k_t$$

$$k_{t+1} + c_t = \left( \left[ \alpha \tau^k + (1-\alpha) \tau^w \right]^{\frac{1-\alpha}{\alpha}} - \left[ \alpha \tau^k + (1-\alpha) \tau^w \right]^{\frac{1}{\alpha}} \right) k_t$$

This is an AK model because we can see that output is linear in the factor  $k$ . This is because when the capital stock increases, the government takes in more tax revenues. It receives more tax revenues because wages are higher (and it taxes wages proportionally), and because the capital stock is higher (and it is taxing capital). The government then feeds this tax revenue into increasing  $\Lambda$ , which increases the return to capital and increases the capital stock more. And, we end up with a linear return in  $k$  which will depend on the taxes.

An increase in either  $\tau^w$  or  $\tau^k$  will increase output and the equilibrium interest rate for the above reason.

(b) The consumers optimization problem is:

$$\max_{c_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t u(c_t) \quad \text{s.t.} \quad c_t + k_{t+1} = (1-\tau^k) r_t k_t + (1-\tau^w) w_t$$

The FOCS are as follows:

$$\beta^t u'(c_t) = \lambda_t \quad \text{for all } t$$

$$\lambda_t = (1-\tau^k) r_{t+1} \lambda_{t+1} \quad \text{for all } t$$

Rewriting this, we have our typical Euler condition:

$$\frac{c_{t+1}}{c_t} = \beta (1-\tau^k) r_{t+1}$$

Plugging in for  $r_{t+1}$ , we have:

$$\frac{c_{t+1}}{c_t} = \beta(1-\tau^k)\alpha[\alpha\tau^k + (1-\alpha)\tau^w]^{\frac{1-\alpha}{\alpha}}$$

We see that a higher  $\tau^w$  will increase the incentive to save because with more tax revenue, the government is able to increase its expenditures and the return to capital rises causing more savings and faster consumption growth. An increase in  $\tau^k$ , however, has an ambiguous affect on the incentive to saving. An increase in the capital tax allows to the government to increase expenditures and the return to capital, but at the same time, the high tax lowers the net return to saving for individuals. Thus, we see it has an ambiguous impact on the growth rate of consumption. The key difference between the two taxes is that the tax on wages essentially acts like a lump sum tax on the individual with no distortions of the savings decision, but the tax on capital does distort the savings decision.

(c) From part (b), we already know that:

$$\gamma = \beta(1-\tau^k)\alpha[\alpha\tau^k + (1-\alpha)\tau^w]^{\frac{1-\alpha}{\alpha}}$$

To solve for  $c/y$ , we take the resource constraint from (a) and rewrite it as:

$$\frac{k_{t+1}}{k_t} + \frac{c_t}{k_t} = \left( [\alpha\tau^k + (1-\alpha)\tau^w]^{\frac{1-\alpha}{\alpha}} - [\alpha\tau^k + (1-\alpha)\tau^w]^{\frac{1}{\alpha}} \right)$$

But, on the balance growth path, it must be the case that capital grows at the same rate as consumption. Thus, we can plug in for the growth rate of capital to get:

$$\begin{aligned} \frac{c_t}{k_t} &= \left( [\alpha\tau^k + (1-\alpha)\tau^w]^{\frac{1-\alpha}{\alpha}} - [\alpha\tau^k + (1-\alpha)\tau^w]^{\frac{1}{\alpha}} \right) - \beta(1-\tau^k)\alpha[\alpha\tau^k + (1-\alpha)\tau^w]^{\frac{1-\alpha}{\alpha}} \\ \frac{c_t}{k_t} &= \left[ 1 - (\alpha\tau^k + (1-\alpha)\tau^w) - \alpha\beta(1-\tau^k) \right] [\alpha\tau^k + (1-\alpha)\tau^w]^{\frac{1-\alpha}{\alpha}} \\ \frac{c_t}{y_t} &= 1 - (\alpha\tau^k + (1-\alpha)\tau^w) - \alpha\beta(1-\tau^k) \end{aligned}$$

(d) The impact of the taxes on  $\gamma$  was already explained in part (b). The impact of the taxes on  $c/y$  are as follows: An increase in the tax on wages clearly reduces  $c/y$ . The higher tax implies a higher return to savings because of more government expenditures, and this means individuals will save more thus driving down the ratio of consumption to output in each period of time. The tax on capital, however, again has the same two opposing effects as before on the incentive to save (and hence the level of consumption). But, we can see that as a whole, an increase in the tax on capital will

push down  $c/y$ . The increase in the interest rate from additional government expenditures will be enough to offset the fact that individuals receive a smaller net return on capital because of the tax. Thus, they will still saving more.

(e) With  $\tau^w = 0$  and  $\tau^k \in [0,1]$ , we have:

$$\gamma = \alpha^{\frac{1}{\alpha}} \beta (1 - \tau^k) [\tau^k]^{\frac{1-\alpha}{\alpha}}$$

$$\frac{c}{y} = 1 - \alpha\beta - \alpha(1 - \beta)\tau^k$$

The value of  $\tau^k$  that maximizes  $c/y$  is zero. The value of  $\tau^k$  that maximizes  $\gamma$  is  $1 - \alpha$

With  $\tau^k = 0$  and  $\tau^w \in [0,1]$ , we have:

$$\frac{c_t}{y_t} = 1 - \alpha\beta - (1 - \alpha)\tau^w$$

$$\gamma = \alpha\beta [(1 - \alpha)\tau^w]^{\frac{1-\alpha}{\alpha}}$$

The value of  $\tau^w$  that maximizes  $c/y$  is zero. The value of  $\tau^w$  that maximizes  $\gamma$  is 1.

The two results are the same for  $c/y$  because we know that on average an increase in either tax reduces the savings of individuals. Thus, the obvious way to increase the relative level of consumption is simply to eliminate the taxes (or set them to zero).

The two results are different for the growth rate of consumption, however, because the tax on capital is distortionary while the tax on wages is not. Since the tax on wages is non-distortionary, the government has an incentive to increase it as high as possible to further increase the return to capital and the growth of consumption. But, with the tax on capital, if the government raises it too high, the additional benefits (from a higher return to capital) will no longer outweigh the fact that individual's incentive to save is distorted downward by the tax because it takes away from their return to savings.

(f) The trade-offs are those just mentioned above. The government will first want to use the tax on wages since it acts essentially like a lump-sum tax and doesn't distort the individual's incentive to save. Then, the government must decide whether it can still increase the growth rate of consumption by using some taxes on capital in addition to already having  $\tau^w = 1$ . But, when  $\tau^w = 1$ , we have:

$$\gamma = \beta(1 - \tau^k) \alpha [\alpha\tau^k + (1 - \alpha)]^{\frac{1-\alpha}{\alpha}}$$

Taking the derivative of this, we can show that the growth rate of consumption is maximized when  $\tau^k = 0$ . Thus, the government only wants to tax wages in this economy.

## 2. Question #2

- (a) With an aggregate technology that exhibits constant returns to scale, we know that there will be a balanced growth path with constant growth. If all countries are on their BGP, then it is hard to reconcile a model of endogenous growth with conditional convergence since everyone may be growing at different rates, and we actually would have divergence in levels.

HOWEVER, it is also plausible to think that not all countries began on their balanced growth path and that countries off the BGP take time to get to the BGP. Then, we could again have convergence dynamics, as maybe countries that began further away from their BGP grow faster as they converge to the BGP.

Additionally, we could also think of endogenous growth models where convergence occurs because of technology transfers from rich to poor countries... Given these two above cases the answer to this question should be UNCERTAIN.

- (b) This statement is false. From the endogenous growth model involving R&D through “Expanding Variety”, we know that monopoly power for those that sell the intermediate goods provides the necessary profits to induce individuals to do R&D in the first place. In this model, more competition can actually reduce the incentive to do R&D and thus reduce the growth rate of the economy (in the competitive equilibrium).

This result comes from the fact that the knowledge is often a ‘non-rival’ good. Once it’s invented (at a cost), its marginal cost of production is often nothing. But at a price of zero (as implied by competition), no one would be able to recoup their initial costly investment to produce the new knowledge. Thus, it is necessary to provide some incentives to do this, and this is often done by granting the inventor some monopoly power, as through a patent.

- (c) The answer is uncertain. From the permanent income hypothesis, we know that the response of contemporaneous consumption to changes in income will depend on how much of the change in income is permanent, and how much is transitory.

When all of the income changes are purely transitory (and permanent, lifetime income is unaffected), then we know that consumption will not respond at all to the changes in income because consumption only responds to changes in permanent income. However, if the movements in income represent pure movements in the expected permanent income, then it is possible that consumption could move one to one with the changes in income.

Finally, you should have noted that the sensitivity of consumption also depends on the degree of insurance (complete vs. incomplete markets) available to the agents in the economy.

- (d) This statement is false. While the neoclassical growth model does reasonably well in accounting for the business cycle movements of output and investment. It does not do a very good job in accounting for the movement in employment. While it does find that employment should be pro-cyclical, the predicted movements in employment (from the RBC model) are much lower than what we see in the data. Moreover, the RBC model does not help explain at all the cyclical variation in the Solow residual (TFP) because it takes it for exogenous.