

MIT 14.11: Problem Set 4  
Fall 2013

Due in class on Tuesday, December 10<sup>th</sup> (no penalty for submission by December 12). If you are working with a partner, you and your partner may turn in a single copy of the problem set. Please show your work and acknowledge any additional resources consulted.

1. Read Chapter 5 in Osborne's An Introduction to Game Theory and complete the following problems:
  - (a) 163.2. Two people select a policy that affects them both by alternately vetoing policies until only one remains. First person 1 vetoes a policy. If more than one policy remains, person 2 then vetoes a policy. If more than one policy still remains, person 1 then vetoes another policy. The process continues until a single policy remains unvetoes. Suppose there are three possible policies,  $X$ ,  $Y$ , and  $Z$ , person 1 prefers  $X$  to  $Y$  to  $Z$ , and person 2 prefers  $Z$  to  $Y$  to  $X$ . Model this situation as an extensive game and find its Nash equilibria.
  - (b) 168.1. Which of the Nash equilibria of the game in Figure 160.1 are subgame perfect?
  - (c) 173.3. Find the subgame perfect equilibria of the game in (a). Does the game have any Nash equilibrium that is not a subgame perfect equilibrium? Is any outcome generated by a Nash equilibrium not generated by any subgame perfect equilibrium? Consider variants of the game in which player 2's preferences may differ from those specified in (a). Are there any preferences for which the outcome in a subgame perfect equilibrium of the game in which player 1 moves first differs from the outcome in a subgame perfect equilibrium in which player 2 moves first?
2. Read Chapter 14, skipping section 14.8. Complete the following problems:
  - (a) 430.1. Find conditions  $k, x, y$  and the discount factor  $\delta$  under which the strategy pair in which each player punishes a deviation by choosing  $D$  for  $k$  periods is a Nash equilibrium of the infinitely repeated game of the *Prisoner's Dilemma*.
  - (b) 431.2. Find conditions on  $x, y$  and the discount factor  $\delta$  under which the strategy pair in which each player uses tit-for-tat is a Nash equilibrium of the infinitely repeated game of the Prisoner's Dilemma. Show in particular that if  $y \geq 2x$  then the strategy pair is not a Nash equilibrium for any value of  $\delta < 1$ .
  - (c) 442.1. Show that in the subgame following the outcome  $(C, D)$  in the first period of an infinitely repeated *Prisoner's Dilemma* with payoffs and  $\delta > \frac{1}{2}$ , player 1 is better off choosing  $C$  in every subsequent period than she is adhering to the grim trigger strategy.
  - (d) 445.1. Consider the infinitely repeated *Prisoner's Dilemma* in which the payoffs of the component game are those given. Show that (tit-for-tat, tit-for-tat) is a subgame perfect equilibrium of this infinitely repeated game with discount factor  $\delta$  if and only if  $y - x = 1$  and  $\delta = \frac{1}{x}$ .

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14.11 Insights from Game Theory into Social Behavior

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