MIT 14.11: Problem Set 3
Fall 2013

Due in class on Tuesday, November 19^{th}. If you are working with a partner, you and your partner may turn in a single copy of the problem set. Please show your work and acknowledge any additional resources consulted.

1. Nash Equilibria and Robustness of Costly Signaling

   The first step in our costly signaling analysis is to recognize that costly signals are supported as Nash equilibria. That is, we perform an analysis using standard game theory tools, without thinking about dynamics just yet.

   Start with the following assumptions:

   - There are two types of senders: *good* and *bad*. $\frac{1}{3}$ of senders are good. There is one type of receivers.
   - There are four levels of signal, $\{1, 2, 3, 4\}$.
   - For good types, sending these signals cost $\{0, 1, 2, 3\}$, respectively. For bad types, sending these signals cost $\{0, 3, 6, 9\}$.
   - Senders get 5 if receivers accept them. Receivers get 5 if accept a good sender, and -5 if accept a bad one.

   (a) Prove that each of the following is a Nash equilibria of this game:
   - Pooling with rejection: good and bad senders send 1, receivers never accept any signal
   - Efficient separating: good send signal 3, bad send 1, and receiver accepts anything ≥ 3
   - Ostentatious separating: good send signal 4, bad send 1, and receiver accepts only signal 4

   (b) Suppose the fraction of good senders increases to 90%. In the pooling equilibrium, would senders still be better off rejecting all senders? Show that there is an alternative equilibrium in which receivers accept all senders. Call this equilibrium "pooling with acceptance." At what proportion of good senders do we switch from pooling with rejection to pooling with acceptance?

   (c) Suppose that when players try to take an action, there is some, ϵ, probability that a different action is taken. For what values of ϵ is the efficient separating equilibrium above no longer equilibrium? In words, explain why it ceases to be equilibria.

2. Robustness of Wright-Fisher and Costly Signaling

   Of course, the static analysis only gets us so far. First of all, because it is based on the assumption that people are choosing their strategies based on conscious calculations. When it comes to their sense of beauty, this is unrealistic. And secondly, the static analysis only tells us that costly signaling is an equilibrium, it doesn't tell us whether we're likely to see it in practice. Maybe we'll see one of the other equilibria. Maybe it won't converge to any of the equilibria. To address these issues, we use dynamics.

In this problem, we provide you with the code to simulate the dynamics of the costly signaling game. Tweak the code to answer the questions below. Start with the assumptions provided in the set-up of question 1.

(a) Run the code as is. Plot the time trend within a single run. Which equilibrium seems most frequent? Summarize the results over many runs. Which equilibrium is most frequent?

(b) Vary the proportion of good senders, setting it equal to $.1, .2, \ldots, .9$. Plot the frequency of the efficient separating equilibrium at each of these values. What happens at the threshold you found in question 1b? Explain how this demonstrates the value of dynamics.

(c) Vary the mutation rate μ, setting it equal to $.05, .1, .15, \ldots, .95$. Plot the frequency of the efficient separating equilibrium at each of these values. What happens at the threshold you found in question 1c?

(d) Remember selection strength, $0 \leq \omega \leq 1$, determines how much the game matters relative to other things that determine fitness. If $\omega = 1$ then fitness is entirely determined by payoffs of this particular game. Vary ω, setting it equal to $1, .9, .8, \ldots, .2, .1$. Plot the frequency of the efficient separating equilibrium at each of these values. Does the efficient separating emerge only for some parameter regions or does it appear robust with respect to selection strength?

3. Robustness of Reinforcement Learning

Now let's answer similar questions for the other dynamic model we reviewed in class, reinforcement learning.

(a) Run the code as is. Plot the time trend within a single run. Which equilibrium seems most frequent? Summarize the results over many runs. Which equilibrium is most frequent?

(b) Vary the proportion of good senders, setting it equal to $.1, .2, \ldots, .9$. Plot the frequency of the efficient separating equilibrium at each of these values. What happens at the threshold you found in question 1b? Explain how this demonstrates the value of dynamics.

(c) Vary the mutation rate μ, setting it equal to $.05, .1, .15, \ldots, .95$. Plot the frequency of the efficient separating equilibrium at each of these values. What happens at the threshold you found in question 1c?

(d) Now, selection strength is determined by two parameters, α and γ. What is the role of these two parameters? Plot the frequency of the efficient separating equilibrium as you vary these parameters.

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