# Lecture 5: Applications of Consumer Theory

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14.121

# Applications of Consumer Theory

Consumer theory is very elegant, but also very abstract.

This lecture: three classic topics that bring consumer theory closer to economic applications:

1. Welfare effects of price changes.
2. Constructing price indices.
3. Aggregating consumer demand.

# Welfare Evaluation of Price Changes

What's effect on consumer's welfare of price change from $p$ to $p'$?

**Examples:** taxes and subsidies, introduction of new good

Obvious answer: change in utility

$$v\left(p', w\right) - v\left(p, w\right)$$

Problem: **which** indirect utility function?

Utility just way of representing preferences.

Different indirect utility functions give different value of $v\left(p', w\right) - v\left(p, w\right)$.

# Money Metric Indirect Utility

Class of indirect utility functions that let us measure effect of price change in **dollar units**: **money metric** indirect utility functions.

Construct from expenditure function:
Start from any indirect utility function $v$, any price vector $\bar{p} \gg 0$, consider

$$e\left(\bar{p}, v\left(p, w\right)\right)$$

$e\left(\bar{p}, u\right)$ is strictly increasing in $u$
$\implies e\left(\bar{p}, v\left(p, w\right)\right)$ is strictly increasing transformation of $v\left(p, w\right)$
$\implies e\left(\bar{p}, v\left(p, w\right)\right)$ is itself an indirect utility function!

Measure welfare effect of price change in dollar terms by

$$e\left(\bar{p}, v\left(p', w\right)\right) - e\left(\bar{p}, v\left(p, w\right)\right)$$

# Money Metric Indirect Utility

$e(\bar{p}, v(p', w)) - e(\bar{p}, v(p, w))$ is independent of choice of $v$: for every $v$,

$$\begin{aligned} e(\bar{p}, v(p, w)) &= \min_{x : u(x) \geq \max_{y \in B(p,w)} u(y)} \bar{p} \cdot x \\ &= \min_{x : x \succsim y \text{ for all } y \in B(p,w)} \bar{p} \cdot x. \end{aligned}$$

$\Longrightarrow$ all money metric indirect utility functions with same $\bar{p}$ are equivalent

Letting $u = v(p, w)$, $u' = v(p', w)$, the difference

$$e(\bar{p}, u') - e(\bar{p}, u)$$

is independent of the utility representation.

How much more money does consumer need to get new utility rather than old utility, when prices given by $\bar{p}$?

# Equivalent Variation and Compensating Variation

Only remaining issue choice of $\bar{p}$.

Two natural choices: initial price vector $p$, new price vector $p'$.

Lead to two best-known ways of measuring welfare effect of price change:
**equivalent variation (EV)** and **compensating variation (CV)**

# Equivalent Variation

EV measures required expenditure change at original prices:

$$EV = e\left(p, u'\right) - e\left(p, u\right) = e\left(p, u'\right) - w$$

EV = amount of money consumer would need to be given **before** price change to make her as well off as would be after price change.

Consumer indifferent between **either** getting EV or facing price change.

EV = amount of money that is "equivalent" to price change.

# Compensating Variation

CV measures required expenditure change at new prices:

$$CV = e\left(p', u'\right) - e\left(p', u\right) = w - e\left(p', u\right)$$

CV = amount of money consumer would need to lose **after** price change to make her as well off as was before price change.

Consumer indifferent between

1. getting **both** (minus) CV and facing price change, and
2. getting neither.

CV = amount of money needed to “compensate” for price change.

# EV vs. CV

EV and CV are different.

Can be ranked if

1. price change affects only one good, and
2. good is either normal or inferior over relevant range of prices.

Follows from connection between EV/CV and Hicksian demand:

$$
\begin{aligned}
EV &= e(p, u') - w \\
&= e(p, u') - e(p', u') \\
&= \int_{p_i'}^{p_i} h_i(p, u')\, dp_i
\end{aligned}
$$

$$
\begin{aligned}
CV &= w - e(p', u) \\
&= e(p, u) - e(p', u) \\
&= \int_{p_i'}^{p_i} h_i(p, u)\, dp_i
\end{aligned}
$$

# EV vs. CV

$$
\begin{aligned}
EV &= \int_{p_i'}^{p_i} h_i\left(p, u'\right) dp_i \\
CV &= \int_{p_i'}^{p_i} h_i\left(p, u\right) dp_i
\end{aligned}
$$

Suppose $p_i > p_i'$ (so $u' > u$).

If good $i$ normal, then $h_i\left(p, u'\right) > h_i\left(p, u\right)$, so $EV > CV$.

If good $i$ inferior, then $h_i\left(p, u'\right) < h_i\left(p, u\right)$, so $EV < CV$.

If no wealth effect for good $i$, then $EV = CV$.

Graphically, EV is area to left of $h_i\left(\cdot, u'\right)$, CV is area to left of $h_i\left(\cdot, u\right)$.

# Marshallian Consumer Surplus

**Marshallian consumer surplus (CS)** = area to left of Marshallian demand curve $x_i(\cdot, w)$:

$$CS = \int_{p_i'}^{p_i} x_i(p, w)\, dp_i$$

For changes in price of one good,

$$\min\{EV, CV\} \leq CS \leq \max\{EV, CV\}$$

Proof.

$x(p, w) = h(p, e(p, u))$, $x(p', w) = h(p', e(p', u'))$, so Marshallian demand curve cuts across region between $h_i(\cdot, u)$ and $h_i(\cdot, u')$. □



If wealth effects small, then $EV$, $CV$, and $CS$ are similar.

# Estimating Welfare from New Goods

CV for new good:

$$\int_{p}^{\infty} h_i\left(p, u\right) dp_i$$

How to estimate demand at very high price?

If price drops to 0 at $\bar{p}$, can estimate

$$\int_{p}^{\bar{p}} h_i\left(p, u\right) dp_i$$

so just have to estimate $\bar{p}$/demand around $\bar{p}$.

See Hausman and Newey (1995, 2011) for recent approaches to estimating welfare from new goods.

# Price Indices

An important application of measures of welfare changes is construction of **price indices**: measures of changes in price level (or **inflation**).

Important for estimating real GDP growth, determining social security payments, negotiating long-term labor contracts, etc..

# Laspeyres and Paasche Indices

Problem is to construct index of price change from period 0 to period 1, where:

- in period 0, see prices $p$ and consumption $x$
- in period 1, see prices $p'$ and consumption $x'$

Laspeyeres index: ratio of price of **original** basket of goods in period 1 to price in period 0:

$$\frac{p' \cdot x}{p \cdot x}$$

Paasche index: ratio of price of **new** basket of goods in period 1 to price in period 0:

$$\frac{p' \cdot x'}{p \cdot x'}$$



Problem: no one cares about $p' \cdot x$ or $p \cdot x'$

# Ideal Indices

Ideal indices are constructed from money metric indirect utility functions.

Measure how much more expensive it gets to attain utility $u$:

$$Ideal\left(u\right) = \frac{e\left(p', u\right)}{e\left(p, u\right)}$$

Ex. $u$ could equal $v\left(p, w\right)$ or $v\left(p', w\right)$

# Biases in Price Indices

Ideal indices let us formalize popular view that Lespeyres "overstates inflation," and Paasche "understates inflation":

$$\text{Laspeyres} = \frac{p' \cdot x}{p \cdot x} = \frac{p' \cdot x}{e\left(p, u\right)} \geq \frac{e\left(p', u\right)}{e\left(p, u\right)} = \mathit{Ideal}\left(u\right)$$

$$\text{Paasche} = \frac{p' \cdot x'}{p \cdot x'} = \frac{e\left(p', u'\right)}{p \cdot x'} \leq \frac{e\left(p', u'\right)}{e\left(p, u'\right)} = \mathit{Ideal}\left(u'\right)$$

Problem is called **substitution bias**: Laspeyres and Paasche don't take into account that, when prices change, consumers substitute to cheaper goods.

# Substitution Bias in Practice

1996 Boskin commission report: CPI overstated by about 1.1 percentage points per year.

Sources of bias:

- **Substitution bias (0.4%):** CPI used Laspeyres index, updated basket of goods very infrequently.
- **Outlet bias (0.1%):** CPI treated different goods at different stores as different. Missed switch to cheaper stores.
- **New goods bias (0.6%):** CPI only tracked changes in price from when new goods were added to basket, not original drop from price $\infty$.

An area of research is getting better estimates of new goods bias. Some economists think 0.6% is way too low.

# Demand Aggregation

Consumer theory concerns behavior of a single consumer.

Often care about **aggregate** behavior of consumers.

Ex. to construct ideal price index for US economy, would need aggregate expenditure function for US population

Does consumer theory also apply to aggregate demand and welfare?

# Demand Aggregation

Three questions:

1. Does aggregate demand depend only on $p$ and **aggregate wealth** $w = \sum_i w^i$, or does distribution of wealth also matter?
2. Does positive theory of individual demand also apply to aggregate demand?
   (Is there a "positive representative consumer"?)
3. Do welfare measures derived from aggregate demand mean anything?
   (Is there a "normative representative consumer"?)

# Aggregate Demand

Suppose there are $I$ consumers.
Consumer $i$ has Marshallian demand $x^i\left(p, w^i\right)$.

**Aggregate demand** $X$ = sum of individual demands:

$$X\left(p, w^1, \ldots, w^I\right) = \sum_{i=1}^{I} x^i\left(p, w^i\right)$$

# Aggregate Demand and Aggregate Wealth

When does aggregate demand depend only on $p$ and $\sum_i w^i$?

When does there exist $\tilde{X} : \mathbb{R}^n_+ \times \mathbb{R} \to \mathbb{R}^n$ such that

$$X\left(p, w^1, \ldots, w^I\right) = \tilde{X}\left(p, \sum_{i=1}^{I} w^i\right) \text{ for all } p, w$$

Clearly, this holds iff every possible redistribution of wealth among consumers leaves aggregate demand unchanged.

Turns out that this holds iff preferences are **quasihomothetic**, a class that generalizes both homothetic and quasilinear preferences.

# Homothetic Preferences

## Definition

Preferences are **homothetic** if, for every $x, y \in \mathbb{R}^n$ and $\alpha > 0$,

$$x \succsim y \Leftrightarrow \alpha x \succsim \alpha y$$

Graphically, preferences are homothetic if slope of indifference curves is constant along rays beginning at the origin.

Homothetic preferences are represented by utility functions that are homogeneous of degree 1:

$$u(\alpha x) = \alpha u(x) \text{ for all } x$$

Demand is homogeneous of degree 1 in income:

$$x(p, \alpha w) = \alpha x(p, w)$$

Have indirect utility function of form:

$$v(p, w) = b(p) w$$

# Quasilinear Preferences

Recall that preferences are quasilinear (in good 1) if admit utility representation of the form

$$u(x) = x_1 + f(x_2, \ldots, x_n)$$

Assuming some income is spend on the numeraire good, have indirect utility function of form:

$$v(p, w) = a(p) + w$$

# Quasihomothetic Preferences

## Definition

Preferences are **quasihomothetic** (or **Gorman form**) if they admit an indirect utility function of the form:

$$v(p, w) = a(p) + b(p) w$$

## Theorem

*Aggregate demand depends only on aggregate wealth iff preferences admit Gorman form indirect utility functions* **with the same function** $b$ **for every consumer***: that is, there exist functions* $a_i : \mathbb{R}^n_+ \to \mathbb{R}$ *and* $b : \mathbb{R}^n_+ \to \mathbb{R}$ *such that, for all* $i$*,*

$$v_i\left(p, w^i\right) = a_i(p) + b(p) w^i.$$

# Quasihomothetic Preferences

## Theorem

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$$v_i\left(p, w^i\right) = a_i\left(p\right) + b\left(p\right) w^i.$$

**Intuition:**

- Aggregate demand depends on aggregate wealth iff not affected by any redistribution of wealth.
- This holds if wealth effects are the same **across individuals** and **across wealth levels**.
- Wealth effects same across wealth levels $\implies$ quasihomothetic preferences.
- Wealth effects same across individuals $\implies$ same function $b$ for everyone.

# Positive Representative Consumer?

Exists if aggregate demand depends only on aggregate wealth: then aggregate demand is same as if all wealth held by consumer 1, so consumer 1 is representative consumer.

Hard to write down conditions much weaker than this that imply existence of representative consumer.

So, representative consumer exists only if not much heterogeneity, especially heterogeneity in wealth effects.

Illustrate with example with well-behaved preferences but no representative consumer.

# Example

Two consumers are buying apples and bananas.
Each consumer has wealth $w = 4$.

Consumer 1 likes apples more.
Consumer 2 likes bananas more.
Neither has much taste for more than two units of same fruit.

$$\begin{aligned} p &= (1,2) : x^1 = (2,1)\,, x^2 = (0,2)\,, x^{agg} = (2,3)\,. \\ \hat{p} &= (2,1) : \hat{x}^1 = (2,0)\,, \hat{x}^2 = (1,2)\,, x^{agg} = (3,2)\,. \end{aligned}$$

Apples are cheap $\Longrightarrow$ more bananas get bought.
Bananas are cheap $\Longrightarrow$ more apples get bought.

Cannot result from optimization by a rational representative consumer (violates WARP).

# Normative Representative Consumer?

A **Bergson-Samuelson social welfare function** is a function $W : \mathbb{R}^I \to \mathbb{R}$ that maps vectors of individual utilities into a "social utility."

When does choosing consumption for each consumer to maximize $W$ (subject to aggregate budget constraint $\sum_i p \cdot x^i \leq \sum_i w^i$) lead to same aggregate consumption that results when each consumer maximizes her utility separately?

Not very often.
One problem: consumption vector that maximizes $W$ depends on choice of utility representation.
If consumer 1 likes apples, consumer 2 likes bananas, and scale up consumer 1's utility by 100, then decentralized aggregate demand is constant, while maximizing $W$ involves buying more apples.

# Representative Consumer with Gorman Form Preferences

With Gorman form preferences (with same $b$), each consumer consumers goods other than numeraire until their marginal utility falls to $b(p)$, puts rest of wealth in numeraire.

$\implies$ decentralized aggregate demand maximizes

$$\sum_{i=1}^{I} a^i(p) + b(p) w^i$$

The same aggregate demand function maximizes **utilitarian** social welfare

$$W\left(u^1, \ldots, u^I\right) = \sum_{i=1}^{I} u^i$$

With Gorman form preferences, decentralized consumer optimization leads to the allocation that maximizes utilitarian social welfare.



Not true more generally.