Lecture 12: Dynamic Choice and Time-Inconsistency

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Dynamic Choice

Most important economic choices are made over time, or affect later decisions.

Standard approach:

- Decision-maker has atemporal preferences over outcomes.
- Makes choice over times to get best outcome.
- Analyze via dynamic programming.

Today: formalize standard approach, also discuss new aspects of choice that arise in dynamic contexts:

- Changing tastes and self-control.
- Preference for flexibility.
- Application: time-inconsistent discounting.

Choice from Menus

Choice over time: choices today affect available options tomorrow.

Ex. consumption-savings.

Model as choice over menus:

- Stage 1: choose **menu** z from set of menus Z.
  - Each menu is a set of outcomes X.
- Stage 2: choose **outcome** $x \in X$.

Ex. Z is set of restaurants, X is set of meals.

The Standard Model of Dynamic Choice

Decision-maker has preferences $\succsim$ over outcomes.

Decision-maker chooses among menus to ultimately get best attainable outcome.

That is, choice over menus maximizes preferences $\dot{\succsim}$ given by

$$z \dot{\succsim} z' \iff \max_{x \in z} u(x) \geq \max_{x' \in z'} u(x'),$$

where $u : X \rightarrow \mathbb{R}$ represents $\succsim$.

Dynamic programming provides techniques for solving these problems.

Example: Restaurants

There are three foods:

$$X = \{Chicken, Steak, Fish\}$$

There are seven restaurants offering different menus:

$$Z = \{\{c\}, \{s\}, \{f\}, \{c, s\}, \{c, f\}, \{s, f\}, \{c, s, f\}\}$$

Suppose consumer's preferences over meals are

$$f \succ c \succ s$$

Then preferences over menus are

$$\{f\} \dot{\sim} \{c, f\} \dot{\sim} \{s, f\} \dot{\sim} \{c, s, f\} \dot{\succ} \{c\} \dot{\sim} \{c, s\} \dot{\succ} \{s\}$$

Example: Consumption-Savings Problem

An outcome is an stream of consumption in every period:

$$x = (c_1, c_2, \ldots)$$

The choice to consume c_1^* in period 1 is a choice of a menu of consumption streams that all have c_1^* in first component:

$$Z = \left\{ (c_1^*, c_2, \ldots), (c_1^*, c_2', \ldots), \ldots \right\}$$

The Standard Model: Characterization

When are preferences over menus consistent with the standard model?
(That is, with choosing $z \in Z$ to maximize $\max_{x \in z} u(x)$ for some $u : X \to \mathbb{R}$.)

Theorem

A rational preference relation over menus $\dot{\succsim}$ is consistent with the standard model iff, for all z, z',

$$z \dot{\succsim} z' \implies z \dot{\sim} z \cup z'$$

Remark: can show that $\{x\} \dot{\succsim} \{y\}$ iff $x \succsim y$.
Thus, preferences over menus pin down preferences over outcomes.

Is the standard model always the right model?

Changing Tastes and Self-Control

Suppose reason why preferences on X are $f \succ c \succ s$ is that consumer wants healthiest meal.

But suppose also that steak is **tempting**, in that consumer always orders steak if it's on the menu.

Then preferences over menus are

$$\{f\} \dot{\sim} \{f, c\} \dot{\succ} \{c\} \dot{\succ} \{s\} \dot{\sim} \{f, s\} \dot{\sim} \{c, s\} \dot{\sim} \{f, c, s\}$$

These preferences are **not** consistent with the standard model: $\{f\} \dot{\succsim} \{s\}$ but $\{f\}$ is not indifferent to $\{f, s\}$.

Implicit assumptions:

- Decision-maker's tastes change between Stage 1 and Stage 2.
- She anticipates this is Stage 1.
- Her behavior in Stage 1 is determined by her tastes in Stage 1.

Temptation and Self-Control

What if consumer is strong-willed, so can resist ordering steak, but that doing so requires exerting costly effort?

Then (if effort cost is small)

$$\{f\} \dot{\sim} \{f,c\} \dot{\succ} \{f,s\} \dot{\sim} \{f,c,s\} \dot{\succ} \{c\} \dot{\succ} \{c,s\} \dot{\succ} \{s\}$$

In general, have

$$z \dot{\succsim} z' \implies z \dot{\succsim} z \cup z' \dot{\succsim} z',$$

but unlike standard model can have strict inequalities.

Gul and Pesendorfer (2001): this **set betweenness** condition (plus the von Neumann-Morgenstern axioms) characterizes preferences over menus with representation of the form

$$U(z) = \max_{x \in z} [u(x) + v(x)] - \max_{y \in z} v(y)$$

Interpretation: u is "true utility", v is "temptation", choice in Stage 2 maximizes $u + v$.

Preference for Flexibility

Another possibility: what if consumer is **unsure** about her future tastes?

Suppose thinks favorite meal likely to be f, but could be c, and even tiny chance of s.

Then could have

$$\{f, c, s\} \dot{\succ} \{f, c\} \dot{\succ} \{f, s\} \dot{\succ} \{f\} \dot{\succ} \{c, s\} \dot{\succ} \{c\} \dot{\succ} \{s\}$$

In general, **preference for flexibility** means

$$z \supseteq z' \implies z \dot{\succsim} z'$$

Preference for Flexibility

Preference for flexibility: $z \supseteq z' \implies z \dot{\succsim} z'$

Another reasonable property:

$$z \dot{\sim} z \cup z' \implies \text{for all } z'',\ z \cup z'' \dot{\sim} z \cup z' \cup z''$$

"If extra flexibility of z' not valuable in presence of z, also not valuable in presence of larger set $z \cup z''$."

Kreps (1979): these properties characterize preferences over menus with representation of the form

$$U(z) = \sum_{s \in S} \left[\max_{x \in z} u(x, s) \right]$$

for some set S and function $u : X \times S \to \mathbb{R}$.

Interpretation: S is set of "subjective states of the world", $u(\cdot, s)$ is "utility in state s".

Example: Time-Consistency in Discounting

For rest of class, explore one very important topic in dynamic choice: discounting streams of additive rewards.

An outcome is a stream of rewards in every period:

$$x = (x_1, x_2, \ldots)$$

Assume value of getting x_t at time t as perceived at time $s \leq t$ is

$$\delta_{t,s} u(x_t)$$

Value of (remainder of) stream of rewards x at time s is

$$\sum_{t=s}^{\infty} \delta_{t,s} u(x_t)$$

Time-Consistency

Question: when is evaluation of stream of rewards from time s onward independence of time at which it is evaluated?

That is, when are preferences over streams of rewards **time-consistent**?

Holds iff tradeoff between utility at time τ and time τ' is the same when evaluated at time t and at time 0:

$$\frac{\delta_{\tau,0}}{\delta_{\tau',0}} = \frac{\delta_{\tau,t}}{\delta_{\tau',t}} \text{ for all } \tau, \tau', t.$$

Normalize $\delta_{t,t} = 1$ for all t. Let $\delta_t \equiv \delta_{t,t-1}$.

Then

$$\frac{\delta_{2,0}}{\delta_{1,0}} = \frac{\delta_{2,1}}{\delta_{1,1}},$$

so

$$\delta_{2,0} = \delta_{2,1}\delta_{1,0} = \delta_2\delta_1.$$

Time-Consistency

By induction, obtain

$$\delta_{t,s} = \prod_{\tau=s+1}^{t} \delta_\tau \text{ for all } s, t.$$

Fix $r > 0$, define Δ_t by

$$e^{-r\Delta_t} = \delta_t.$$

Then

$$\delta_{t,s} = \exp\left(-r \sum_{\tau=s+1}^{t} \Delta_\tau\right).$$

Conclusion: time-consistent discounting equivalent to maximizing exponentially discounted rewards with constant discount rate, allowing real time between periods to vary.

If periods are evenly spaced, get standard exponential discounting: $\delta_t = \delta$ for all t, so

$$\sum_{t=0}^{\infty} \delta_{t,0} u(x_t) = \sum_{t=0}^{\infty} \delta^t u(x_t).$$

Time-Inconsistent Discounting

Experimental evidence suggests that some subjects exhibit **decreasing impatience**: $\delta_{t+1,s}/\delta_{t,s}$ is decreasing in s.

Ex. Would you prefer \$99 today or \$100 tomorrow?
Would you prefer \$99 next Wednesday or \$100 next Thursday?

Aside: Doesn't necessarily violate time-consistency, as can have $\delta_{nextThursday} > \delta_{thisThursday}$.
But if ask again next Wednesday, then want the money then.

Quasi-Hyperbolic Discounting

What kind of discounting can model this time-inconsistent behavior?

Many possibilities, most influential is so-called **quasi-hyperbolic discounting:**

$$\delta_{t,s} = \begin{cases} 1 \text{ if } t = s \\ \beta\delta^{t-s} \text{ is } t > s \end{cases}$$

where $\beta \in [0, 1]$, $\delta \in (0, 1)$.

$\beta = 1$: standard exponential discounting.

$\beta < 1$: **present-bias**
Compare future periods with **each other** using exponential discounting, but hit all future periods with an extra β.

Quasi-Hyperbolic Discounting: Example

Suppose $\beta = 0.9$, $\delta = 1$.

Choosing today:

- $99 today worth 99, $100 tomorrow worth 90.
- $99 next Wednesday worth 89.1, $100 next Thursday worth 90.

Choosing next Wednesday:

- $99 today worth 99, $100 tomorrow worth 90.

Quasi-Hyperbolic Discounting

How will someone wil quasi-hyperbolic preferences actually behave?

Three possibilities:

1. Full commitment solution.
2. Naive planning solution.
3. Sophisticated (or "consistent") planning solution.

Quasi-Hyperbolic Discounting: Full Commitment

If decision-maker today can find a way to commit to future consumption path, time-inconsistency is inconsequential.

This helps explain various commitment devices.

Assuming for simplicity that wealth is storable at 0 interest, problem is

$$\max_{(x_t)_{t=0}^{\infty}} \sum_{t=0}^{\infty} \delta_{t,0} u(x_t)$$

subject to

$$\sum_{t=0}^{\infty} x_t \le w.$$

FOC:

$$\frac{u'(x_t^*)}{u'(x_{t+1}^*)} = \frac{\delta_{t+1,0}}{\delta_{t,0}}$$

End up consuming more in period 0 relative to $\beta = 1$ case, otherwise completely standard.

Quasi-Hyperbolic Discounting: No Commitment

What if commitment impossible?

Two possibilities:

- Consumer realizes tastes will change (sophisticated solution).
- Consumer doesn't realize tastes will change (naive solution).

Quasi-Hyperbolic Discounting: Naive Solution

At time 0, consumer solves full commitment problem as above, consumes $x_0^*(w_0)$, saves $w_1 = w_0 - x_0^*(w_0)$.

At time 1, consumer does **not** go along with plan and consume $x_1^*(w_0)$.

Instead, solves full commitment problem with initial wealth w_1, consumes $x_0^*(w_1)$.

Due to quasi-hyperbolic discounting, $x_0^*(w_1) > x_1^*(w_0)$. Consumes more than she was supposed to according to original plan.

Same thing happens at time 2, etc..

Note: solve model **forward** from time 0.

Quasi-Hyperbolic Discounting: Sophisticated Solution

At time 0, consumer must think about what her “time-1 self” will do with whatever wealth she leaves her.

Time-0 self and time-1 self must also think about what time-2 self will do, and so on.

The decision problem becomes a **game** among the multiple selves of the decision-maker.

Must be analyzed with an **equilibrium** concept.

Intuitively, must solve model **backward**: think about what last self will do with whatever wealth she's left with, then work backward.

You'll learn how to do this in 122.