

14.122 Problem Set #3

1. Prove the following theorem (known as the Debreu-Fan-Glicksburg Theorem), which establishes sufficient conditions for the existence of a *pure strategy* Nash equilibrium in certain games with continuous action spaces.

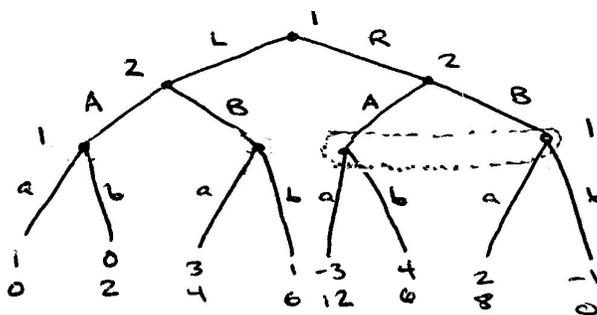
Theorem 1 *Let G be a normal form game. Suppose that the strategy sets S_i are nonempty, compact, convex subsets of Euclidean space and that the payoff functions u_i are continuous in s and quasiconcave in s_i . Then G has a pure strategy Nash equilibrium.*

(Hint: the proof is very similar to that of Nash's Theorem.)

2. Let G be a two player normal form game with strategy spaces A_1 and A_2 . A mixed strategy Nash equilibrium σ^* is said to be *totally mixed* if $\sigma_i^*(a_i) > 0$ for all $a_i \in A_i$. Show that if G has more than one totally mixed Nash equilibrium then it must have infinitely many.

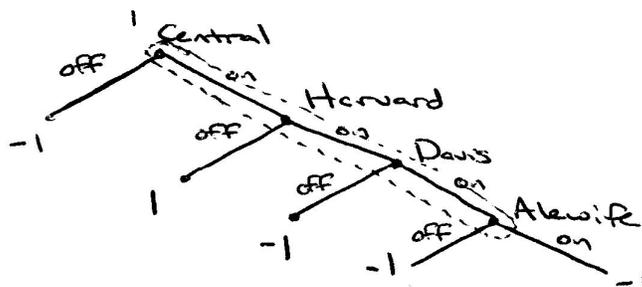
Can you give an example of a game which has more than one but finitely mixed strategy Nash equilibria (which are not pure strategy NE)?

3. (a) Write out a complete description of the set of nodes, information sets, action, successor and payoff functions, etc. in the game below.



(b) Write out the normal representation of the game and find all of the pure strategy Nash equilibria.

4. Let G be the game shown below where a drunk student is trying to get from Kendall Square to Harvard Square, but can not recall each time the doors of the subway open how many stops (if any) have passed.



(a) A game is said to have perfect recall if whenever x and x' are in the same information set neither is a predecessor of the other, and whenever $x'' \in h(x')$, x is a predecessor of x' and the same player i moves at x and x' there exists a node \hat{x} (possibly equal to x) such that \hat{x} is in the same information set as x , \hat{x} is a predecessor of x'' and the action taken at x along the path to x' is the same as the action taken at \hat{x} along the path to x'' . (See Fudenberg-Tirole p. 81.) Show formally that the game above does not have perfect recall.

(b) How many pure strategies does player 1 have in this game. What is the set of mixed (behavior) strategies available to him?

(c) What payoff does player 1 receive from each of his pure strategies? What payoff does player 1 receive from mixing and getting off with probability p ? What is the optimum choice of p ?

(d) I've repeated many times in this class that players utility functions are a linear function of the probabilities with which they mix over their pure strategies strategies. Why does this idea not extend to games of imperfect recall in the way in which you might have expected? What might the proper generalization be? (Feel free to not think about this game much if you think it confuses you and will make you forget the basic point that payoffs are linear in mixing probabilities.)