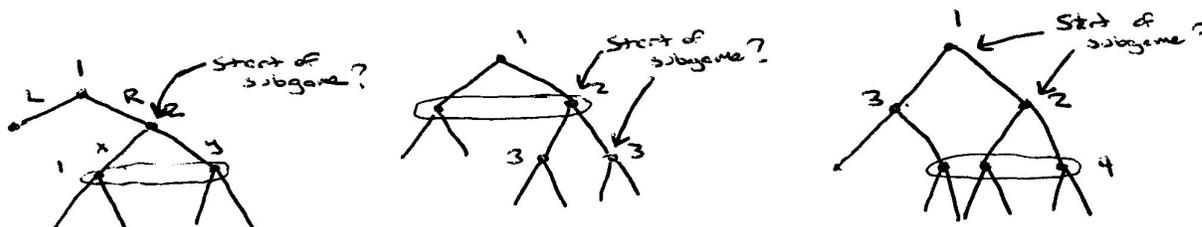
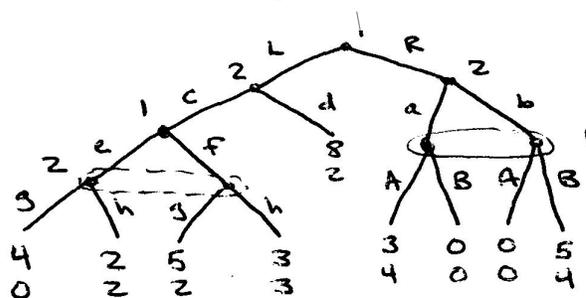


14.122 Problem Set #4

1. For each of the extensive forms below say whether the set of successors of the indicated nodes are or are not subgames.



2. Find all of the subgame perfect equilibria of the following extensive form game and give the payoffs obtained by the players in each of them.



3. Suppose two companies, one American and one Japanese, can each produce a homogeneous good for sale in the U.S. market at marginal cost c , $0 \leq c < 1$. However, the Japanese firm must also pay a tariff of t per unit sold. Assume that the inverse demand is given by $P(Q) = 1 - Q$ where Q is the total quantity produced by the two firms.

(a) Find the Nash equilibrium of the Cournot-like game where the firms simultaneously choose quantities q_A and q_J (with t fixed). How high must the tariff be to eliminate all Japanese imports?

(b) Suppose the U.S. government chooses t before the firms choose quantities, and that the objective of the government is to maximize the sum of consumer surplus, the profit of the domestic firm and tax revenues. What tariff level is chosen in a subgame perfect equilibrium?

(c) Find a Nash equilibrium of the game above in which the U.S. government achieves a lower utility level than it does in the subgame perfect equilibrium.

4. Consider the following model of monetary policy. In the first stage, the firms in the economy form an expectation π^e of what the inflation rate will be and sign wage contracts, make investments, etc. on the basis of these expectations. Next, the Federal Reserve takes actions which determine the actual inflation rate π . Unexpectedly high inflation stimulates the economy and increases aggregate output Y , but any departures of inflation from expectations cause losses to the firms. In particular, assume that $Y = Y_0 + a \log(1 + \pi - \pi^e)$ and that the firms' profits are $-(\pi - \pi^e)^2$. Suppose also that the utility of the Federal Reserve Chairman is given by $U_{\text{fed}} = Y - c\pi^2$.

Find the inflation level in the subgame perfect equilibrium of this game. Does the subgame perfect equilibrium involve rational expectations (*i.e.* is $\pi^e = \pi$)? Comment on how the equilibrium inflation rate varies with a and c ?

5. Suppose that two players play each other for two periods. In the first period they play the game at left below, and in the second period they play the game at right. There is no discounting between periods. Players observe the *action* their opponent took in the first period before choosing their second period actions.

(a) For $x \leq 2$ and $y \leq 6$, find a subgame perfect equilibrium in which player 1 receives a payoff of 10.

(b) For $x = 5$ and $y = 3$, find a subgame perfect equilibrium in which player 2 receives a payoff of 10.

(c) For $x = y = 4$ show that there is no subgame perfect equilibrium in which (U, L) is played in the first period.

	L	R	
U	2, 2	-10, x	
D	y , 0	0, 0	

	L	R
U	8, 4	0, 0
D	0, 0	4, 8

4. Consider the one player infinite horizon game in which player 1 chooses an action $a_t \in \{0, 1\}$ at $t = 1, 2, 3, \dots$, and receives a payoff of 1 if $\lim_{t \rightarrow \infty} a_t > \frac{1}{2}$ and a payoff of zero otherwise.

(a) Show that the payoffs in this game are not continuous at infinity.

(b) Find a strategy profile which is not a subgame perfect equilibrium despite the fact that the player can not change his strategy at a single information set h and improve his payoff conditional on that information set being reached.