

### Taxing externalities with measurable pollution

Atmosphere:

$$A = \sum_i a^i x_0^i \quad (1)$$

The only route for externalities is through  $A$ . Note  $A$  is the same for all consumers. Note linearity is not important. This could be done with a vector of different atmospheres, e.g., for different locations.

Pareto optimality assuming linear technology with fixed producer prices  $p$ :

$$\begin{aligned} \text{Max} \quad & \sum_h \alpha^h u^h [x_0^h, x^h, A] \\ \text{s.t.} \quad & \sum_h (p_0 x_0^h + p \cdot x^h) = R \end{aligned} \quad (2)$$

First order conditions with respect to  $x_i^h, x_0^h$ :

$$\alpha^h \frac{\partial u^h}{\partial x_i} = \lambda p_i \quad h=1,2,\dots,H; i=1,2,\dots,N \quad (3)$$

$$\alpha^h \frac{\partial u^h}{\partial x_0} + a^h \sum_k \alpha^k \frac{\partial u^k}{\partial A} = \lambda p_0 \quad (4)$$

Substituting from (3) in (4)

$$\frac{\partial u^h / \partial x_0^h}{\partial u^h / \partial x_1^h} = \frac{p_0}{p_1} - a^h \sum_k \frac{\partial u^k / \partial A}{\partial u^k / \partial x_1^k} \quad (5)$$

- I. If we can measure the pollution contribution,  $a^h x_0^h$ , we can decentralize the PO by pricing pollution, although prices might need to vary by person.

The consumer problem becomes:

$$\begin{aligned} \text{Max } & u^h [x_0^h, x^h, A] \\ \text{s.t. } & p_0 x_0 + p \cdot x + t^h a^h x_0 = I^h \end{aligned} \quad (6)$$

First order conditions

$$\frac{\partial u^h / \partial x_0^h + a^h \partial u^h / \partial A}{\partial u^h / \partial x_1^h} = \frac{p_0 + t^h a^h}{p_1} \quad (7)$$

This will support the PO provided

$$\frac{t^h}{p_1} = \frac{\partial u^h / \partial A}{\partial u^h / \partial x_1^h} - \sum_k \frac{\partial u^k / \partial A}{\partial u^k / \partial x_1^k} \quad (8)$$

If individuals ignore their own feedback to the atmosphere, individual choice now has FOC:

$$\frac{\partial u^h / \partial x_0^h}{\partial u^h / \partial x_1^h} = \frac{p_0 + t^h a^h}{p_1} \quad (9)$$

This allows support for the PO with uniform taxes

$$\frac{t}{p_1} = - \sum_k \frac{\partial u^k / \partial A}{\partial u^k / \partial x_1^k} \quad (10)$$

Note this extends to a vector of (local) atmospheres and more than one externality generating good, provided pricing distinguishes each atmosphere.

II. Alternatively, assuming consumers ignore the feedback on self through  $A$ , decentralization can be approached by taxing good zero.

$$\begin{aligned} \text{Max } & u^h [x_0^h, x^h, A] \\ \text{s.t. } & (p_0 + t^h)x_0^h + p \cdot x^h = I^h \end{aligned} \tag{11}$$

First order condition:

$$\frac{\partial u^h / \partial x_0^h}{\partial u^h / \partial x_i^h} = \frac{p_0 + t^h}{p_i} \tag{12}$$

$$t^h = a^h p_1 \sum_k \frac{\partial u^k / \partial A}{\partial u^k / \partial x_1^k} \tag{13}$$

This relies on the lack of choice in how the good is consumed, with different choices resulting in different levels of pollution