

Handout on stock market model

2 periods, 1 good per period, multiplicative uncertainty
only assets: safe real bond, shares

Notation:

x_0	consumption in period 0
x_{1s}	consumption in period 1 in state s
π_s^h	subjective probability of household h for state s
b	bonds
q	price of bonds
D_s^f	dividends of firm f in state s
q^f	price of all shares in firm f
θ_f^h	fraction of firm f owned by household h after purchase
θ_f^{eh}	fraction of firm f in initial endowment of household h
k^f	input of firm f

Consumer choice:

$$\begin{aligned}
 & \max_{x_0, x_1} \sum_s \pi_s^h u^h(x_0, x_{1s}) \\
 \text{s.t.} \quad & x_0 + qb + \sum_f q^f \theta_f^h = e_0^h + \sum_f q^f \theta_f^{eh} \\
 & x_{1s} = b + \sum_f \theta_f^h D_s^f + e_s^h \quad \forall s
 \end{aligned} \tag{1}$$

Dividend payment:

$$\begin{aligned}
 y_s^f &= a_s^f g^f(k^f) \\
 D_s^f &= a_s^f g^f(k^f) - \frac{k^f}{q}
 \end{aligned} \tag{2}$$

Firm maximization of stock market value:

$$\begin{aligned} \max_k \quad & q^f \\ \text{s.t.} \quad & \text{"competitive perceptions"} \end{aligned} \tag{3}$$

$$(q^f + k^f) \frac{g'^f(k^f)}{g^f(k^f)} = 1 \tag{4}$$

Market clearance

$$\sum_h x_0^h = \sum_h e_0^h - \sum_f k^f \tag{5}$$

Conditions for constrained Pareto optimality:

$$\begin{aligned} \max \quad & \sum_s \pi_s^1 u^1(x_0^1, x_{1s}^1) \\ \text{s.t.} \quad & \sum_s \pi_s^h u^h(x_0^h, x_{1s}^h) = \bar{v}^h, \quad h = 2, \dots, H \\ & \sum_h x_0^h = \sum_h e_0^h - \sum_f k^f \\ & x_{1s}^h = e_{1s}^h + \sum_f \mu_f^h y_s^f + z^h \\ & \sum_h z^h = 0 \\ & \sum_h \mu_f^h = 1 \end{aligned} \tag{6}$$