Handout contrasting stock market (SM) and complete contingent commodity (CCC) models

2 periods, 1 good per period, multiplicative uncertainty
no-bankruptcy and no-short-sales constraints not binding
second period has S states

SM: only assets: safe real bond, shares

CCC: all commodities

Notation

b	bonds
q	price of bonds
D_s^f	dividends of firm f in state s
q^f	price of all shares in firm f
θ_f^h	fraction of firm f purchased by household h
θ_f^{eh}	fraction of firm f in initial endowment of household h
p_0	price of good in period 0
p_{1s}	price of good in state s in period 1
Q^f	profit of firm f

Consumer choice:

SM:

$$\max_{x_0, x_1} \sum_s \pi_s^h u^h\left(x_0, x_{1s}\right)$$
$$\text{s.t. } x_0 + qb + \sum_f q^f \theta_f^h = e_0^h + \sum_f q^f \theta_f^{eh}$$
$$x_{1s} = b + \sum_f \theta_f^h D_s^f + e_s^h \quad \forall s$$

CCC:

$$\max_{x_0, x_1} \sum_s \pi_s^h u^h\left(x_0, x_{1s}\right)$$
$$\text{s.t. } p_0 x_0 + \sum\nolimits_s p_{1s} x_{1s} = \sum_f \theta_f^{eh} Q^f + \sum\nolimits_s p_{1s} e_s^h \quad (1)$$

SM has $S+1$ budget constraints, while CCC has 1.

$$D_s^f = a_s^f g^f\left(k^f\right) - \frac{k^f}{q} \qquad\qquad Q^f = \sum\nolimits_s p_{1s} a_s^f g^f\left(k^f\right) - p_0 k^f \quad (2)$$

Firm choice:

$$\max_k \; q^f \quad \text{s.t. "competitive perceptions"} \qquad\qquad \max_k \; Q^f \quad \text{s.t. "competitive perceptions"} \quad (3)$$

First order conditions:

$$\left(q^f + k^f\right)\frac{g'^f\left(k^f\right)}{g^f\left(k^f\right)} = 1 \qquad \sum_s p_{1s} a_s^f g'^f\left(k^f\right) = p_0 \tag{4}$$

Market clearance:

$$\sum_h x_0^h = \sum_h e_0^h - \sum_f k^f \qquad \sum_h x_0^h = \sum_h e_0^h - \sum_f k^f \tag{5a}$$

$$\sum_h \theta_f^h = 1 \quad \text{f=1,...,F} \qquad \sum_h x_{1s}^h = \sum_h e_{1s}^h + \sum_f a_s^f g^f\left(k^f\right) \quad \text{s=1,...,S} \tag{5b}$$

$$\sum_h q b^h = \sum_f k^f \tag{5c}$$

Note that Walras Law gives (5c).

Constrained Pareto optimality:

$$\begin{aligned}
\max \quad & \sum_s \pi_s^1 u^1\left(x_0^1, x_{1s}^1\right) \\
\text{s.t.} \quad & \sum_s \pi_s^h u^h\left(x_0^h, x_{1s}^h\right) = v^{-h}, \quad h = 2,\ldots,H \\
& \sum_h x_0^h = \sum_h e_0^h - \sum_f k^f \\
& x_{1s}^h = e_{1s}^h + \sum_f \mu_f^h a_s^f g^f\left(k^f\right) + z^h \\
& \sum_h \mu_f^h = 1 \\
& \sum_h z^h = 0
\end{aligned}$$

Pareto optimality:

$$\begin{aligned}
\max \quad & \sum_s \pi_s^1 u^1\left(x_0^1, x_{1s}^1\right) \\
\text{s.t.} \quad & \sum_s \pi_s^h u^h\left(x_0^h, x_{1s}^h\right) = v^{-h}, \quad h = 2,\ldots,H \\
& \sum_h x_0^h = \sum_h e_0^h - \sum_f k^f \\
& \sum_h x_{1s}^h = \sum_h e_{1s}^h + \sum_f a_s^f g^f\left(k^f\right)
\end{aligned} \tag{6}$$