

## 14.123 Microeconomics III—Problem Set 2

Muhamet Yildiz

**Instructions.** You are encouraged to work in groups, but everybody must write their own solutions. Each question is 33 points. Good Luck!

1. Ann is an expected utility maximizer, but she does not know her preferences, which she can learn by costly contemplation. To model this situation, take  $S = [0, 1]$ , and let  $Z \subseteq \mathbb{R}$  be a finite set of consequences with at least two elements. Assume that Ann's von Neumann utility function is

$$u(z) = z \quad \forall z \in Z,$$

and her belief on  $S$  is represented by uniform distribution. For any  $n$  and some fixed  $c > 0$ , by spending  $cn$  utils, Ann can obtain a partition

$$P_n = \{[0, 1/2^n], (1/2^n, 2/2^n), \dots, (k/2^n, (k+1)/2^n), \dots, [(2^n - 1)/2^n, 1]\}$$

and observe the cell  $I_n(s) \in P_n$  in which the true state  $s$  lies. After the observation, she assigns uniform distribution on  $I_n(s)$  and can choose an act  $f : S \rightarrow Z$  under the new belief. Her eventual payoff is  $u(f(s)) - cn$ . Now imagine that, given any two acts  $f$  and  $g$ , Ann first chooses  $n$  and, after observing the cell in which  $s$  lies, she chooses one of the acts  $f$  and  $g$ . She does so in order to maximize her expected payoff minus the cost  $cn$ , knowing all along that she will choose one of the acts  $f$  and  $g$  optimally based on her observation. Write  $f \succeq_s g$  if Ann may end up choosing  $f$  when the true state happens to be  $s$ . Check which of the postulates P1-P5 of Savage is satisfied by  $\succeq_s$  for any fixed  $s$ .

2. Ann is a risk-averse expected utility maximizer with an increasing utility function  $u : \mathbb{R} \rightarrow \mathbb{R}$  and with an initial wealth of  $w_0$ . She is indifferent between accepting and rejecting a lottery that gives \$1 (gain) with probability  $p = 0.6$  and  $-\$1$  (loss) with probability  $(1 - p)$ .

- (a) Find the smallest  $G$  for which Ann is willing to accept a lottery that gives  $\$G$  (gain) with probability  $1/2$  and  $-\$L = -\$100,000$  (loss) with probability  $1/2$  consistent with above information. That is, find

$$G^* = \min \{G | u(G + w_0) + u(-L + w_0) \geq u(w_0), u \in U\}$$

where  $U$  is the set of utility functions described above.

- (b) What would be your answer to (a) if you also knew that Ann has a constant absolute risk aversion over  $[w_0 - 100, w_0 + 100]$ .
- (c) What would be your answer to (a) if you also knew that Ann has a constant absolute risk aversion (everywhere).
- (d) What would be your answer to (a) if you also knew that Ann has a constant relative risk aversion (everywhere).

3. Consider identical individuals  $i = 1, \dots, n$  with constant absolute risk aversion  $\alpha > 0$  and dates  $t = 1, \dots, T$ . Each  $i$  has an asset that pays

$$Y_i = X_{i1} + \dots + X_{iT}$$

at time  $T$  where the random variables  $X_{it}$ ,  $i = 1, \dots, n$ ,  $t = 1, \dots, T$ , are all independently and identically distributed with  $N(\mu, \sigma^2)$ . Each  $X_{it}$  becomes publicly observable at time  $t$ . There is also risk-neutral company who wants to buy these assets. At the beginning of each time  $t$ , before  $(X_{1t}, \dots, X_{nt})$  becomes observable, the company offers a price  $P_{it}$  to each individual  $i$  who has not sold his asset to the company. If an individual  $i$  accepts the offer, the company owns the asset of  $i$  and pays  $P_{it}$  to the individual  $i$  at time  $T$ . (Note that individual  $i$  gets  $Y_i$  if he holds on to his asset and  $P_{it}$  if he sells his asset at some time  $t$ , getting the money at time  $T$  in both cases.)

- (a) For any  $t$ , any realization  $(X_{i1}, \dots, X_{i(t-1)})$  and any  $i$ , determine the lowest price  $P_{it}$  at which  $i$  is willing to sell his asset if he still owns it. Should the company offer  $P_{it}$ ?
- (b) Consider a new identical individual (with constant risk-aversion parameter  $\alpha$ ). How much is he willing to pay for a  $1/n$ th share of the company (getting  $1/n$ th of the net proceedings of the company at  $T$ )?
- (c) What does the answer to part (b) change if the company is barred from trading at date  $t = 1$ ?
- (d) Briefly discuss your results.

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14.123 Microeconomic Theory III  
Spring 2015

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