14.123 Microeconomics III—Problem Set 3

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Instructions. You are encouraged to work in groups, but everybody must write their own solutions. Each question is 25 points. Good Luck!

1. Ann is a risk-averse expected utility maximizer with an increasing utility function $u : \mathbb{R} \to \mathbb{R}$ and with an initial wealth of w_0. She is indifferent between accepting and rejecting a lottery that gives \$1 (gain) with probability $p = 0.6$ and $-\$1$ (loss) with probability $(1 - p)$.

   (a) Find the smallest G for which Ann is willing to accept a lottery that gives $\$G$ (gain) with probability $1/2$ and $-\$L = -\$100,000$ (loss) with probability $1/2$ consistent with above information. That is, find

   $$G^* = \min \{G | u(G + w_0) + u(-L + w_0) \geq u(w_0), u \in U\}$$

   where U is the set of utility functions described above.

   (b) What would be your answer to (a) if you also knew that Ann has a constant absolute risk aversion over $[w_0 - 100, w_0 + 100]$.

   (c) What would be your answer to (a) if you also knew that Ann has a constant absolute risk aversion (everywhere).

   (d) What would be your answer to (a) if you also knew that Ann has a constant relative risk aversion (everywhere).

2. Bob has just retired and has w_0 dollars. His utility from a consumption stream $(c_0, c_1, \ldots)$ is

   $$\sum_{t=0}^{n} \delta^t u(c_t),$$

   where $u : R \to R$ is a von Neumann-Morgenstern utility function with constant relative risk aversion $\rho > 1$. For each t, he dies in between periods t and $t + 1$ with probability p, in which case he gets 0 utility.

   (a) Take $n = 1$, and find the optimal consumption stream c^* with $c_0^* + c_1^* \leq w_0$.

   (b) Take $n = \infty$, and find the optimal consumption stream c^* with $c_0^* + c_1^* + \cdots \leq w_0$.

   (c) What would be your answer to part (b) if $\rho = 1$?

3. Solve Problem 2, assuming instead that Bob can get r_t from each dollars saved at t, i.e., w dollars saved at t becomes wr_t dollars at $t + 1$, where (r_t) is i.i.d. with $r_t > 0$ and $\delta E\left[r_t^{1-\rho}\right] \in (0, 1)$.

4. This question is about a game, called "Deal or No Deal". The monetary unit is M\$, which means million dollars. The players are a Banker and a Contestant. There are n cases: 1, 2, ..., n. One of the cases contains 1M\$ and all the other cases contain

zero M$. All cases are equally likely to contain the 1M$ prize (with probability $1/n$). Contestant owns Case 1. Banker offers a price p_0, and Contestant accepts or rejects the offer. If she accepts, then Banker buys the content of Case 1 for price p_0, ending the game. (Contestant gets p_0 M$ and Banker gets the content of the case, minus p_0 M$.) If she rejects the offer, then we open Case 2, revealing the content to both players. Banker again offers a price p_1, and Contestant accepts or rejects the offer. If she accepts, then Banker buys the content of Case 1 for price p_1; otherwise we open the next case (Case 3), and this goes on until all the cases $2, \ldots, n$ are opened. When all the cases $2, \ldots, n$ are opened, the game ends with Contestant owning the content of Case 1 and Banker owning zero. The utility of owning x M$ is x for the Banker and $x^{1/\alpha}$ for the Contestant, where $\alpha > 1$.

(a) Assuming α is commonly known, apply backward induction to find a subgame-perfect equilibrium.

(b) Take $n = 3$. Now assume that Banker does not know α, i.e., α is private information of Contestant, and $\Pr(1/2^\alpha \leq x) = 2x$ for any $x \leq 1/2$. Consider a strategy of the Contestant with cutoffs $\hat{\alpha}_0(p_0)$ and $\hat{\alpha}_1(p_1)$ such that Contestant accepts the first price p_0 iff $\alpha \geq \hat{\alpha}_0(p_0)$ and, in the case the game proceeds to the next stage, she accepts the second price p_1 iff $\alpha \geq \hat{\alpha}_1(p_1)$. Find the necessary and sufficient conditions on $\hat{\alpha}_0(p_0)$ and $\hat{\alpha}_1(p_1)$ under which the above strategy is played by the contestant in a sequential equilibrium. (You need to find two equations, one contains only $\hat{\alpha}_0(p_0)$ and p_0 and the other contains only $\hat{\alpha}(p_1)$ and p_1 as variables.)