

PSET 3 - SOLUTIONS

Question 1

Part (a)

We know that the DM is an expected utility maximizer. Therefore her preferences \succeq over acts are completely described by her utility function $u : [0, \infty) \rightarrow \mathbb{R}$ and probability $P \in \Delta(S)$. In fact, for two acts $f, g : S \rightarrow [0, \infty)$,

$$f \succeq g \Leftrightarrow U(f) := \sum_{s \in S} P(s)u(f(s)) \geq \sum_{s \in S} P(s)u(g(s)) =: U(g).$$

We already know what is u : $u(c) = \sqrt{c}$ for $c \in [0, \infty)$. So we only need to find P . Write f_s for the Arrow-Debreu security corresponding to state s : for all state s'

$$f_s(s') = \begin{cases} 1 & \text{if } s' = s, \\ 0 & \text{else.} \end{cases}$$

Observe that $U(f_s) = P(s)$, so to find $P(s)$ it is enough to measure $U(f_s)$. We know how the DM ranks f_s against any $c \in [0, \infty)$, that is, we can say whether the DM prefers f_s to a constant act. This means in particular that for every s we have enough data to find $c_s \in [0, \infty)$ such that $f_s \sim c_s$. But $f_s \sim c_s$ implies that

$$P(s) = U(f_s) = U(c_s) = \sqrt{c_s}.$$

So we have found P , and we can conclude that for an arbitrary act f

$$U(f) = \sum_{s \in S} \sqrt{c_s} f(s).$$

Remark. This exercise has a “Anscombe-Aumann flavour.” In the Savage approach, to elicit u and P , first we estimate the probability (chopping the state space using P6), and then we use the probability to measure utility (applying mixture space theorem). The Anscombe-Aumann

methodology goes the other way round. First utility is measured using objective lotteries (applying mixture space theorem). Then certainty equivalents are used to measure the probability, which is what we do here. Even if objective lotteries are slightly mysterious objects, the Anscombe-Aumann method is way more flexible than Savage's, and nowadays most of the works in decision theory adopts it (e.g., ambiguity).

Part (b)

Note that any act can be written as a non-negative linear combination of Arrow-Debreu securities, and any non-negative linear combination of Arrow-Debreu securities is an act. Therefore the portfolio problem for the DM is:

$$\text{maximize } U(f) \text{ over } f \in F \text{ subject to } \sum_{s \in S} f(s)p_s \leq M.$$

The solution to this problem is: for all s

$$f(s) = \frac{c_s/p_s^2}{\sum_{s' \in S} c_{s'}/p_{s'}} M.$$

Question 2

See solution to question 1 pset 2 2014.

Question 3

See solution to question 3 pset 1 2014.

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