14.123 Microeconomics III—Problem Set 4

Muhamet Yildiz

Instructions. You are encouraged to work in groups, but everybody must write their own solutions. Each question is 25 points. Good Luck!

1. Ann is ambiguity-averse and has constant absolute risk aversion α. There is a stock that pays y per unit where $y \sim N(\mu, \sigma^2)$ where $\mu \in [\underline{\mu}, \bar{\mu}]$ is ambiguous. (She maximizes $\min_\mu E[u|\mu]$ where u is a CARA utility with α.) Ann can buy or sell any amount of stock.

   (a) Compute the demand of Ann for the stock as a function of price P of the stock.

   (b) Suppose there are n copies of Ann and Y total units of stock. Find the market clearing price.

   (c) Answer the above questions by assuming instead that Ann is an expected utility maximizer with μ uniformly distributed on $[\underline{\mu}, \bar{\mu}]$.

   (d) Briefly discuss your answers.

2. There is an asset that pays y where y has c.d.f. F. Ann is a rank-dependent expected utility maximizer with linear utility (i.e. she is "risk-neutral") and with probability weighting function $w : [0,1] \to [0,1]$ where $w(x) = x^\beta$ for some $\beta > 0$.

   (a) How much is she willing to pay for the asset if y is exponentially distributed. (You do not need to compute the integral.)

   (b) Show that for any y with $0 < F(y) < 1$, there exist β_1 and β_2 such that Ann is willing to pay less than y whenever $\beta < \beta_1$ and willing to pay more than y whenever $\beta > \beta_2$.

3. Ann is as in Prospect Theory. She has a reference dependent utility function

$$u(x|x_0) = v(x - x_0)$$

   where

$$v(y) = \begin{cases} y & \text{if } y \geq 0 \\ \lambda y & \text{otherwise} \end{cases}$$

   for some $\lambda \geq 1$ and the reference point x_0 is her initial wealth. Her probability weighting function is identity mapping (i.e. she does not distort the probabilities). For every initial wealth level, she is indifferent between accepting and rejecting a lottery that gives \$1 (gain) with probability $p = 0.6$ and $-\$1$ (loss) with probability $(1-p)$.

   (a) Find the smallest G for which Ann is willing to accept a lottery that gives $\$G$ (gain) with probability 1/2 and $-\$L = -\$100,000$ (loss) with probability 1/2 consistent with above information.

   (b) Briefly discuss your finding by comparing to your answers to Problem 1 in Problem Set 3.

4. Ann and Bob are negotiating over dividing a dollar, as in alternating-offer bargaining. Ann makes an offer $(x, 1-x)$ and Bob accepts or rejects the offer. If the offer is accepted, the game ends, and Ann gets x and Bob gets $1-x$. If the offer is rejected, Bob makes an offer $(y, 1-y)$ day, and Ann decides whether to accept the offer. This goes on with alternating proposers until an offer is accepted. Assume that Ann and Bob both have hyperbolic discounting: according to the self at time t, getting x dollars at s has a value of $\delta(s-t)$ for some decreasing function $\delta : \mathbb{R} \to (0,1)$. Assuming that the players are sophisticated (i.e. the payoffs specified above are common knowledge), compute the subgame-perfect equilibrium.