B Holmstrom
14.124
Spring 2003

HOMEWORK 3 – due March 11, before class.

NOTE: Only questions 1-3 will be graded (and counted as part of the homework grade). The other problems are extra and will be discussed in the recitation.

Question 1. Problem 14. B. 4 (**only** parts a-c)

Question 2

In the linear incentive model, $w(x)=\alpha x+\beta$, the cost function of the agent is $c(e, s) = \frac{1}{2}(e + s)^2 - s$, where e is time put into working for the principal and s is time spent with family and friends. The negative sign in front of s comes from the fact that the agent values time spent with family and friends. The agent's utility function is exponential with risk aversion coefficient $r = 2$. Output from work is $x = e + \theta$, where θ is Normally distributed with mean zero and variance 4.

a. Suppose the incentive coefficient $\alpha = .3$ How much time will the agent spend on family and friends?

b. Suppose the principal can set $s = 0$ by asking the agent to come to the office each day. What is the optimal choice of α in that case?

c. Suppose the agent's best alternative to working for the principal is spending time with family and friends. What will the principal have to pay the agent as a fixed salary (β) in part b?

Question 3

This question is about optimal contracting in a principal-agent model with uniform distribution function. Let the production technology be described by $x = e + \theta$. The agent chooses "effort" e privately (the principal cannot observe e). The principal observes x, which therefore can be used as a basis for contracting. Assume θ is distributed uniformly on [0,1]. The agent's utility function takes the form $U(m,e) = u(m) - c(e)$, over money m and effort e. Here u is concave, c is convex, and both are increasing functions. The principal is risk neutral.

a. What is the first-best contract (ie the contract when e can be observed and contracted on)?

b. Show that the first-best outcome can be achieved even when e is not observed by the principal. That is, show that in the special case of a uniform distribution the principal-agent problem does not entail any welfare loss relative to the first-best.

QUESTION 4. (extra).

An agent has utility function u(x) = sqrt(x) – c, where sqrt stands for square root, x is money and c is the choice (and cost) of effort. Effort cannot be observed. If the agent chooses effort c = 1.5, the outcome is 200 half the time and 0 the rest of the time. If the agent chooses c = 2.5, the outcome is 200 with probability .7 and zero with probability .3. These two c-values are the agent's only feasible choices. The agent's best market alternative is to work for a pay of w = 9 at the cost c = 0. The principal is risk neutral and owns the technology.

a. Suppose the principal wants to implement c = 1.5. What contract should the principal offer to the agent? (Note that any contract has to pay non-negative wages in both states, because of the square root utility function).

b. Suppose the principal wants to implement c = 2.5. What is the principal's best contract offer in this case? Comparing answers a and b, what will be the best contract for the principal?

c. Change the problem slightly and assume that if the agent chooses c = 1.5, the two possible outcomes are: 200 with probability .5 and – 5 with probability .5. If the agent chooses c = 2.5, the outcomes and probabilities are as specified before. Argue that the principal can do no worse in this case than in the original case. Can the principal do better? (Hint: Argue first that whenever the outcome is – 5, the agent should be paid 0.)