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14.124
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Homework 4 – due March 18, before class

Note: Only problems 1, 2 and 3 will be graded.

1. 14.C.8

2. Consider the used car market model discussed in class, but with warranties. The value of the car to the seller is v = \$10000μ, where μ is the probability that the car will not break down during the next 20000 miles (the car will break down at most once). The seller knows μ. The buyer thinks μ is uniformly distributed between .50 and 1.00. This implies that the value v is uniformly distributed between \$5000 and \$10000, as was the case in the original example. Recall that without warranties, the maximum trading range was [\$5000, \$7000], because \$7000 was the highest price the buyer could profitably offer.

Consider warranties of the form: If the car breaks down during the first 20000 miles the seller pays buyer \$1000. Let p be the price offered for the car cum warranty package.

a. Given an offer p, find the range of sellers (expressed in terms of μ), who will accept the offer p.

b. Given the answer in part a, what is the buyer's expected value conditional on the offer p being accepted? What is the maximum p-value that the buyer is willing to offer?

c. Are there offers p that will result in a range of trades that exceeds the maximum range [\$5000, \$7000] in the original model? If so, what is the price that gives the largest range?

   Hint: first calculate a range of p values which sellers with a v valuation in [\$5000, \$7000] will find acceptable. Then calculate what cars will trade when the buyer makes the highest possible p-offer, as calculated in b.

3. Consider the following regulation problem. Let x be output, c be marginal cost of output and R be net return of production. The firm's net profit before regulatory payments is given by

$$R = x - cx, \text{ for } 0 \leq x \leq 1.$$

Note that x has to lie in the unit interval and that output has been normalized to equal revenue. Assume that the regulator does not know the firm's cost coefficient c, which can take on n values: $0 \leq c_1 \leq ... \leq c_n \leq 1$. The regulator assesses probability $p_i \geq 0$ to each of the i possible marginal cost levels. The firm knows its marginal cost and chooses x. The regulator can tax the firm based on its realized revenue.

a. Set up the program that solves for the regulator's optimal policy, expressed as a direct revelation mechanism. Assume that the regulator's objective is to extract as much surplus from the firm as possible, subject to the constraint that the firm can quit if profits are negative (and pay nothing).

b. Draw a diagram that shows how the firm chooses its best response as a function of its cost. Prove that the firm's output choice x is a non-decreasing function of marginal cost.

c. Show that the optimal policy takes the simple form: a fixed charge T for producing anything (hence, all firms with $c \leq T$ produce and those with $c > T$ do not). It suffices that you give an argument for $n = 3$.

4. Consider a screening model of the following sort. The agent produces output q at a cost $c(q,\beta) = q\beta$, where q is output and β is a cost parameter. The principal, who cannot observe either c or β, offers to pay the agent the amount $p(q)$ if the agent produces output q. Given this incentive scheme, an agent with cost parameter β responds by producing the amount $q(\beta)$.

What kind of payment scheme $p(q)$ should the principal choose in order to implement the response function $q(\beta) = 1/\beta^2$?