14.127 Behavioral Economics. Lecture 13

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0.1 Prospect Theory and Asset Pricing

- Barberis, Huang, Santos, QJE 2001

- $V = \max E\left[\sum_{t\geq 0} \rho^t \frac{c^{1-\gamma}}{1-\gamma} + b_t \rho^{t+1} v\left(x_{t+1}, s_t, z_t\right)\right]$

- S_t – dollar amount invested in stocks

- $x_{t+1} = s_t \left(R_t - R_{rf}\right)$ where R_t - stock return, R_{rf} - risk-free rate

- Z_t – historical benchmark level for risky assets, $z_t = \frac{Z_t}{S_t}$

- the agent is "in the domain of gains" iff $z_t < 1$

- $b_t = b_0 \bar{c}_t^{-\gamma}$ in order to have same rate of growth for both terms in the parentheses

- Dynamics $z_{t+1} = \eta z_t \frac{\bar{R}}{R_{t+1}} + (1 - \eta)$

- If $z > 1$ then $v = x_{t+1} \begin{cases} 1 & \text{if } x_{t+1} > 0 \\ \lambda(z_t) & \text{if } x_{t+1} \leq 0 \end{cases}$ with $\lambda(z_t)$ increasing in z_t

- $\lambda(z) = \lambda + k(z - 1)$

- If $z < 1$ then $v = s_t \begin{cases} R_{t+1} - R_{rf} & \text{if } R_{t+1} \geq z_t R_{rf} \\ R_{rf}(z_t - 1) + \lambda\left(R_{t+1} - z_t R_{rf}\right) & \text{if } R_{t+1} < z_t R_{rf} \end{cases}$

- Dividend $\ln \frac{D_{t+1}}{D_t} = g_0 + \sigma_D \varepsilon_{t+1}$ and consumptions $\ln \frac{C_{t+1}}{C_t} = g_0 + \sigma_C \eta_{t+1}$ with $\binom{\varepsilon}{\eta} \sim N\left(\binom{0}{0}, \begin{pmatrix} 1 & w \\ w & 1 \end{pmatrix}\right)$

- State variable z_t, $f(z_t) = \frac{P_t}{D_t}$, f to be determined

- Solution

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{f(z_{t+1}) + 1}{f(z_t)} \frac{D_{t+1}}{D_t}$$

- To get Euler equation vary $\delta C_t = -\varepsilon$, $\delta C_{t+1} = \varepsilon R_{rf}$. Since $\delta V = 0$ so

$$\begin{aligned} 0 = \delta V &= \rho^t u'(C_t)\,\delta C_t + \rho^{t+1} u'(C_{t+1})\,\delta C_{t+1} \\ &\rho^t C_t^{-\gamma}(-\varepsilon) + \rho^{t+1} C_{t+1}^{-\gamma} R_{rf}\varepsilon \end{aligned}$$

and

$$1 = E\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \rho R_{rf}\right)$$

- Perturbation with risky asset $\delta C_t = -\varepsilon$, $\delta C_{t+1} = \varepsilon R_{t+1}$. Since $\delta V = 0$ so

$$0 = \delta V = \rho^t u'(C_t)\, \delta C_t + E\left[\rho^{t+1} u'(C_{t+1})\, \delta C_{t+1}\right] + b_t \rho^{t+1} E \frac{v(x_{t+1}, s_t, z_t)}{s_t}$$
$$= \rho^t C_t^{-\gamma}(-\varepsilon) + E\left[\rho^{t+1} C_{t+1}^{-\gamma} R_{t+1} \varepsilon\right] + b_t \rho^{t+1} E \frac{v(x_{t+1}, s_t, z_t)}{s_t} \varepsilon$$

and

$$1 = E\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \rho R_{t+1}\right) + b_0 \rho E \frac{v(x_{t+1}, s_t, z_t)}{s_t}$$
$$= E\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \rho R_{t+1}\right) + b_0 \rho E \hat{v}(R_{t+1}, z_t)$$

- Thus

$$1 = E\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\rho\frac{f\left(z_{t+1}\right)+1}{f\left(z_t\right)}\frac{D_{t+1}}{D_t}\right)+b_0\rho E\hat{v}\left(\frac{f\left(z_{t+1}\right)+1}{f\left(z_t\right)}\frac{D_{t+1}}{D_t}, z_t\right)$$

- $f = \frac{P}{D}$ is decreasing in z

- Problem: make this tractable (like Veronesi and Santos made tractable version of Campbell-Cochrane)

- See tables 3,4,6 of the paper [separate file]

1 Data (see handout)

- Fama and French disprove CAPM (see handout)
  - Propose a rational three factor ($n = 3$) model $Er^i_{t+1} = r_{rf} + \sum_{k=1}^{n} \beta_{ik}\pi_k$ where π_k is risk premium on factor k, β_{ik} – beta of asset i on factor k
  - One of the factors HML = high minus low book to market
  - Half of the finance papers have now their factors in the regressions
- Fama and French on momentum (see handout) – this arbitrage requires constant rebalancing of portfolio and may be killed by transaction costs; it also involves small illiquid stocks

- Forward discount puzzle $s_{t+1} - s_t = r^{\text{foreign}} - r^{\text{domestic}}$, but if you run the regression you get negative values (s_t is foreign exchange rate)

- Same puzzle for bonds

- Warning: after many puzzles are discovered the effects become usually much less strong:

  - so either they are arbitraged away

  - or they were due to data mining.

- Some puzzles are robust, e.g. the bond yield puzzle

2 Bubbles

- Kindleberger "Manias, Panics and Crashes"

- Bubble feeds on inflow of less and less sophisticated investors

- People who predict crash are repeatedly disconfirmed, hence public trusts more those that correctly predicted growth

- Limits to arbitrage:

  - even rational looking hedge funds did ride the bubble.

- some hedge funds did short but if they did it (or move out of the market) too early they were closed – other hedge funds were doing much better

- No good quantitative analysis of bubbles in behavioral finance

- P/E ratios increased after introduction of 401k accounts. 401k increased demand for stocks

- Persistence of very high growth rates

- See slides [a listing of bubbles, graph of NASDAQ, bubble in 1998-1999, and CISCO's three year annualized growth in EPS]