14.127 Lecture 5

Xavier Gabaix

March 4, 2004

0.1 Welfare and noise. A compliment

- Two firms produce roughly identical goods

- Demand of firm 1 is

$$D_1 = P\left(q - p_1 + \sigma\varepsilon_1 > q - p_2 + \sigma\varepsilon_2\right)$$

where $\varepsilon_1, \varepsilon_2$ are iid $N(0,1)$.

- Thus

$$\begin{aligned} D_1 &= P\left(p_2 - p_1 > \sigma\left(\varepsilon_1 - \varepsilon_2\right)\right) = P\left(p_2 - p_1 > \sigma\sqrt{2}\eta\right) \\ &= P\left(\frac{p_2 - p_1}{\sigma\sqrt{2}} > \eta\right) = \bar{\Phi}\left(\frac{p_2 - p_1}{\sigma\sqrt{2}}\right) \end{aligned}$$

where η is $N(0,1)$ and $\bar{\Phi} = 1 - \Phi$, with Φ cdf of $N(0,1)$

- Unlike in $\varepsilon \equiv 0$ case, here the demand is not dramatically elastic

- Slope of demand at the symmetric equilibrium $p_1 = p_2$

$$-\frac{\partial}{\partial p_1} D_1 = -\frac{\partial}{\partial p_1} \bar{\Phi}\left(\frac{p_2 - p_1}{\sigma\sqrt{2}}\right) = \phi\left(\frac{p_2 - p_1}{\sigma\sqrt{2}}\right)\frac{1}{\sigma\sqrt{2}}$$
$$= \phi(0)\frac{1}{\sigma\sqrt{2}}$$

  and "modified" elasticity

$$\eta = -\frac{1}{D_1}\frac{\partial}{\partial p_1} D_1 = \phi(0)\frac{1}{\sigma\sqrt{2}} = \frac{\sqrt{\pi}}{\sigma}$$

  because $D_1 = \frac{1}{2}$.

- When $\sigma \longrightarrow 0$ then $\eta \longrightarrow \infty$. Even though the "true" elasticity is ∞ the measured elasticity is lower $\eta < \eta^{\text{true}}$.

- Open question: how to correct that bias?

0.2 How to measure the quantity of noise σ?

- - Give people n mutual funds and ask them to pick their preferred and next preferred fund.
  - Assume that all those funds have the same value $q_A = q_B$
  - People do

$$\max q_i - p_i + \sigma\varepsilon_i = s_i$$

  - Call A - the best fund, B - the second best fund, $s_A \geq s_B \geq$all other funds.
  - Increase p_A by Δp. At some point the consumer is indifferent between A and B.

$$q_A - p_A + \sigma\varepsilon_A - \Delta p = q_B - p_B + \sigma\varepsilon_B$$

- If $p_A = p_B$ then

$$\Delta p = \sigma \left(\varepsilon_A - \varepsilon_B \right)$$

or

$$\Delta p = \sigma \left(\varepsilon_{(1:n)} - \varepsilon_{(2:n)} \right)$$

- **Proposition.** For large n

$$\Delta p = B_n \sigma$$

where B_n is the parameter of Gumbel attraction,

$$B_n = \frac{1}{n f \left(\bar{F} \left(\frac{1}{n} \right) \right)}$$

0.3 Could the fees be due to search costs?

- Ali Hortacsu and Chad Syverson, QJE 2004, forthcoming.
  - Suppose you have $x = \$200,000$ and you keep it for 10 years.
  - You pay 1.5%/year and thus lose $200,000 \times 1.5\% = 3,000$ a year.

- Competing explanation – people don't know that two index mutual fund are the same thing.

0.4 Open questions

- What are the regulatory implications of consumer confusion?

- Where does confusion $\sigma\varepsilon_i$ comes from? For instance, provide a cognitive model that gives a microfoundation for this "noise"

- Find a model that predicts the level of the confusion σ? e.g., in the mutual fund market, give a model that predicts the reasonable order of magnitude.

- Find a model that predicts how σ varies with experience?

- How do firms increase/create confusion σ?

- Empirically, how could we distinguish whether profits come from true product differentiation, search costs, or confusion noise?

- Devise a novel empirical strategy to measure an effect related to the material of lectures 3 to 5.

0.5 Competition and confusion

- **Proposition.** Firms have an incentive to increase the confusion. The effect is stronger, the stronger is competition.

- Example – cell phone pricing.

- Symmetry of firms is important here. If there is a firm that is much better than others, then it wants to have very low σ to signal this.

- Proof.
  - Consider n identical firms and symmetric equilibria.

$$D_1 = P\left(q - p_1 + \sigma_1\varepsilon_1 + V\left(\sigma_1\right) > \max q - p_i + \sigma_i\varepsilon_i + V\left(\sigma_i\right)\right)$$
$$= P\left(q - p_1 + \sigma_1\varepsilon_1 + V\left(\sigma_1\right) > \max q - p^* + \sigma^*\varepsilon_i V\left(\sigma^*\right)\right)$$

    where $V\left(\sigma_i\right)$ is the utility of complexity σ_i (equated with confusion).
  - Denote $M_{n-1} = \max_{i=2,\ldots,n} \varepsilon_i$. In equilibrium

$$D_1 = P\left(p^* - p_1 + \sigma_1\varepsilon_1 + V\left(\sigma_1\right) - V\left(\sigma^*\right) > \sigma^* M_{n-1}\right)$$
$$= P\left(\varepsilon_1 > \frac{\sigma^*}{\sigma_1}M_{n-1} - \frac{p^* - p_1 + V\left(\sigma_1\right) - V\left(\sigma^*\right)}{\sigma_1}\right) = E\bar{F}\left(c\right)$$

  - At the equilibrium, $p_1 = p^*$, $\sigma_1 = \sigma^*$, and by symmetry

$$D_1 = \frac{1}{n}.$$

– Let us check it to develop flexibility with tricks of the trade. First note that

$$P(M_{n-1} < x) = P((\forall i)\, \varepsilon_i < x) \\ = F(x)^{n-1}$$

∗ Density $g_{n-1}(x) = G'_{n-1}(x) = (n-1) F^{n-2}(x) f(x)$.

∗ Now

$$1 - D_1 = E\left(1 - \bar{F}(M_{n-1})\right) = E(F(M_{n-1})) = \int F(x)\, g_{n-1}(x)\, dx \\ = \int F(x)(n-1) F^{n-2}(x) f(x)\, dx = \int (n-1) F^{n-1}(x) f(x \\ = (n-1)\left[\frac{F^n(x)}{n}\right]_{-\infty}^{+\infty} = \frac{n-1}{n} = 1 - \frac{1}{n}$$

∗ Thus $D_1 = \frac{1}{n}$.

- Heuristic remark. $E\left(\bar{F}\left(M_{n-1}\right)\right) = \frac{1}{n}$. Hence $M_n = A_n + B_n \eta$, where $A_n >> B_n$ are Gumbel attraction constants. Thus $M_n \simeq A_n$. So,

$$M_n \simeq \bar{F}\left(\frac{1}{n}\right) \simeq A_n$$

– The profit $\pi_1 = \max_{p_1,\sigma_1} E\bar{F}\left(\frac{\sigma^*}{\sigma_1}M_{n-1} - \frac{p^*-p_1+V(\sigma_1)-V(\sigma^*)}{\sigma_1}\right)(p_1 - c_1)$

– From FOC and envelope theorem

$$0 = \frac{d}{d\sigma_1}\pi_1 = (p_1 - c_1)\frac{\partial}{\partial\sigma_1}D_1$$

∗ Note that

$$\frac{\partial}{\partial\sigma_1}D_1 = E\left(-f(c_n)\left(-\frac{\sigma^* M_{n-1}}{\sigma_1^2} - \frac{-(p^* - p_1 + V(\sigma_1) - V(\sigma^*))}{\sigma_1^2} + \right.\right.$$

∗ In equilibrium, $c_n = M_{n-1}$, hence

$$0 = \frac{\partial}{\partial\sigma_1}D_1 = E\left(-f(M_{n-1})\left(-\frac{M_{n-1}}{\sigma^*} - \frac{V'(\sigma_1)}{\sigma^*}\right)\right)$$
$$= E\left(f(M_{n-1})\frac{M_{n-1}}{\sigma^*}\right) + E\left(f(M_{n-1})\frac{V'(\sigma_1)}{\sigma^*}\right)$$

- Hence
$$V'(\sigma_1) = -\frac{E\left(f\left(M_{n-1}\right)M_{n-1}\right)}{Ef\left(M_{n-1}\right)} \equiv -d_n$$

- Consider some simple cases
  - Uniform distribution $d_n = 1 - \frac{2}{n}$
  - Gumbel $d_n = \ln n + A$
  - Gaussian $d_n \sim \sqrt{\ln n}$

- In those cases, $V'(\sigma_1) < 0$.

- Thus we have excess complexity.

- What happens as competition grows while $n \to \infty$?
  - Take the utility of noise to be $V(\sigma) = 1 - \frac{1}{2\chi}(\sigma - \sigma^{**})^2$.
  - Then $V'(\sigma) = \frac{-1}{\chi}(\sigma - \sigma^{**}) = -d_n$, and consequently

$$\sigma = \sigma^{**} + \chi d_n$$

- Hence, if competition grows, the problem gets exarcerbated.

0.5.1 Open question. The market for advice works very badly. Why?

- – The fund manager wants to sell their own funds.

  – Advisor charges you 1% per year for advice: he gives you stories each month that suggest some kind of trade. Otherwise, he could lose client.

1 Marketing - Introduction

- Why high prices of add-ons and low prices of printers or cars?

- Often the high add-ons fees are paid by the poor not rich who might be argued have low marginal value of money, e.g. use of credit card to facilitate transactions.

- Many goods have "shrouded attributes" that some people don't anticipate when deciding on a purchase.

- Consider buying a printer.
  - Some consumers only look at printer prices.
  - They don't look up the cost of cartridges.
- Shrouded add-ons will have large mark-ups.
  - Even in competitive markets.
  - Even when demand is price-elastic.
  - Even when advertising is free.

2 Shrouded attributes

- Consider a bank that sells two kinds of services.
- For price p a consumer can open an account.
- If consumer violates minimum she pays fee $\widehat{p}$.
- WLOG assume that the true cost to the bank is zero.
- Consumer benefits V from violating the minimum.

- Consumer alternatively may reduce expenditure to generate liquidity V.

	Do not violate minimum	Violate minimum
Spend normally	0	$V - \widehat{p}$
Spend less	$V - e > 0$	$V - e - \widehat{p}$

2.1 Sophisticated consumer

- Sophisticates anticipate the fee $\widehat{p}$.

- They choose to spend less, with payoff $V - e$

- ...or to violate the minimum, with payoff $V - \widehat{p}$

2.2 Naive consumer

- Naive consumers do not fully anticipate the fee $\widehat{p}$.

- Naive consumers may completely overlook the aftermarket or they may mistakenly believe that $\widehat{p} < e$.

- Naive consumers will not spend at a reduced rate.

- Naive consumer must choose between foregoing payoff V or paying fee $\widehat{p}$.

2.3 Summary of the model

- Sophisticates will buy the add-on iff $V - \widehat{p} \geq V - e$.

- Naives will buy the add-on iff $V - \widehat{p} \geq 0$.

- $D(x_i)$ is the probability that a consumer opens an account at bank i.

- For sophisticated consumer

$$D_i = P\left(q - p_i + \max\left(V - e, V - \widehat{p_i}\right) + \sigma\varepsilon_i > q - p^* + \max\left(V - e, V - \widehat{p}\right.\right.$$
$$= P\left(\sigma\varepsilon_i + x > \sigma \max_{j \neq i} \varepsilon_j\right)$$

- Let α – fraction of rational (sophisticated) consumers, $1-\alpha$ – fraction of irrational (naive) consumers

- Profit earned from rational consumers

$$\pi = \alpha \left(p + \widehat{p} \mathbf{1}_{\widehat{p} \leq e} \right) D \left(-p + \max \left(V - e, V - \widehat{p} \right) + p^{*} - \max \left(V - e, V - p^{*} \right.\right.$$

- Profit earned on irrational consumers

$$(1-\alpha) \left(p + \widehat{p} \mathbf{1}_{\widehat{p} \leq V} \right) D \left(-p + p^{*} \right)$$

Proposition. Call $\alpha^{\dagger} = 1 - \frac{e}{V}$ and $\mu = \frac{D(0)}{D'(0)}$.

- If $\alpha < \alpha^{\dagger}$,equilibrium prices are

$$p = -(1-\alpha)V + \mu$$
$$\widehat{p} = V$$

  and only naive agents consume the add-on.

- If $\alpha \geq \alpha^{\dagger}$, prices are

$$p = -e + \mu$$
$$\widehat{p} = e$$

  and all agents consume the add-on.

Corollary. If $\alpha < \alpha^\dagger$, then the equilibrium profits equal

$$\begin{aligned}\pi &= \alpha p D(0) + (1-\alpha)(p+\widehat{p}) D(0) \\ &= (p + (1-\alpha)\widehat{p}) D(0) = \mu D(0) = \frac{\mu}{n}\end{aligned}$$

- Firms set high mark-ups in the add-on market.

- If there aren't many sophisticates, the add-on mark-ups will be inefficiently high: $\widehat{p} = V > e$.

- High mark-ups for the add-on are offset by low or negative mark-ups on the base good.

- To see this, assume market is competitive, so $\mu \simeq 0$.

  - Loss leader base good: $p^* \approx -(1 - \alpha) V < 0$.

- Examples: printers, hotels, banks, credit card teaser, mortgage teaser, cell phone, etc...

- The shrouded market becomes the profit-center because at least some consumers don't anticipate the shrouded add-on market and won't respond to a price cut in the shrouded market.

- Interpretations

  - bounded rationality, people don't see small print.

  - overconfidence – people believe they will not fail prey to small print penalties.