14.127 Behavioral Economics. Lecture 9

Xavier Gabaix

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1 Self Control Problems

1.1 Hyperbolic discounting

- Do you want a small cookie u_s now ($t_0 = 0$) or a big cookie u_b later ($t_1 =$1 week)?

- Many people prefer $(u_s, 0)$ to (u_b, t_1)

- Denote by $\Delta(t)$ the discount factor applied to time t

- Then
$$\Delta(0)\, u_s > \Delta(t_1)\, u_b.$$

- At the same time many people prefer (u_s, t) to $(u_b, t + t_1)$ where t =1 year, and $t_1 = 1$ day.
$$\Delta(t)\, u_s < \Delta(t + t_1)\, u_b.$$

- Thus,
$$\frac{\Delta(t + t_1)}{\Delta(t)} > \frac{u_s}{u_b} > \frac{\Delta(t_1)}{\Delta(0)}$$

- Denote

$$\psi(t) = \frac{\Delta(t+t_1)}{\Delta(t)}$$

and note

$$\psi(t) > \psi(0)$$

- $$\frac{\psi(t)-1}{t_1} = \frac{1}{\Delta(t)}\frac{\Delta(t+t_1)-\Delta(t)}{t_1} \simeq \frac{1}{\Delta(t)}\Delta'(t)$$

- Thus $\frac{\Delta'(t)}{\Delta(t)}$ is increasing.

- Let us write $\Delta(t) = e^{-\int_0^t \rho(s) ds}$

- Then $\frac{\Delta'(t)}{\Delta(t)} = \frac{d}{dt} \ln \Delta(t) = \frac{d}{dt} \left(-\int_0^t \rho(s) \, ds \right) = -\rho(t)$

- Standard exponential model $\Delta(t) = e^{-\rho s}$, $\rho(s) = \overline{\rho}$

- Empirical evidence points to $\rho(t)$ decreasing

- In comparison of today and tomorrow emotions are silent, in comparison of 1000 days from now and 1001 days cognition takes over.

- Maybe people compare ratios: 1 in t =1000 days vs X_t in $t+1$ =1001 days. For indifference something like $X_t \simeq \frac{1001}{1000}$ is plausible.

$$X_t \simeq 1 + \frac{a}{t}$$

for large t. Clearly $X_t \to 1$ as $t \to \infty$.

- But, $X_t = \frac{\Delta(t)}{\Delta(t+h)} = e^{\int_t^{t+h} \rho(s)ds}$. Thus $X_t \to 1$ iff $\rho(t) \to 0$.

- If $X(t) = 1 + \frac{a}{t}$, then $1 + \frac{a}{t} = X(t) = e^{\int_t^{t+h} \rho(s)ds} \simeq 1 + \int_t^{t+h} \rho(s)\, ds$.

- Thus $\rho(t) \simeq \frac{ah}{t}$ for large t.

- Thus $\int_1^t \rho(s)\, ds \simeq ah \int_1^t \frac{1}{s} ds = ah \ln t = a' \ln t$

- Postulate $\Delta(t) = e^{-a' \ln(t+1)} = \frac{1}{(1+t)^{a'}}$.

- That's why this is called hyperbolic discounting

- Quasi-hyperbolic approximation (Phelps and Pollack 1968, Laibson 1997)

$$\Delta(t) = \begin{cases} 1 & \text{for } t = 0 \\ \beta\delta^t & \text{for } t \geq 1 \end{cases}$$

- Typically, $\beta \leq 1$.

- Now,

$$\frac{\Delta(1)}{\Delta(0)} = \beta\delta < \delta = \frac{\Delta(2)}{\Delta(1)}$$

- This function is tractable. It does not get $X_t \to 1$ though.

1.2 Open question

- What is $t = 1$? For cookie it might be 1 hour. For small money it might be 1 week. For macro consumption it is one quarter. Empirically, $\delta \simeq .98$ in yearly units, and $\beta \simeq .6$ is usually found for all time units.

- What determines β? Clearly, the appeal of the good seems to matter. A nice, moist cookie may have a lower β, while a fairly stale plain bagel may have a β close to 1.

1.3 Dynamic inconsistency

- Example. Do the task (taxes) at $t \in \{0, 1, 2\}$ at a cost $c_0 = 1$, $c_1 = 1.5$, $c_2 = 2.5$. Take $\beta = \frac{1}{2}$ and $\delta = 1$.

  - Take Self 0 (the decision maker at time 0). Disutility of doing the task at 0 is 1, at 1 is $\frac{3}{4}$, at time 2 is 1.25. So, Self 0 would to the task to be done at $t = 1$.

  - Self 1 compares time 1 cost of 1.5 with time 2 cost of 1.25 and prefers the task to be done at time 2.

  - Self 2 does the task at the cost 2.5.

- Proposition. If the decision criterion at t is $\max \sum_{s\geq 0} \Delta(s)\, u(c_{t+s})$ then there is dynamic inconsistency unless there exists a constant η such that $\Delta(s) = \Delta(0)\, \eta^t$.

- Proof (sketch). Take $t = 0$ and choose c_0.
  - Self 0 planned $c_1, c_2, \ldots$ maximizes $\max \sum_{s\geq 1} \Delta(s)\, u(c_s)$ over $c_1, c_2, \ldots$ satysfying a budget constraint.
  - Self 1 maximizes $\max \sum_{s\geq 1} \Delta(s-1)\, u(c_s)$ subject to the same budget constraint
  - For the choices to be the same, there must be a constant η s.t. $(\Delta(s))_{s\geq 1} = \eta\, (\Delta(s-1))_{s\geq 1}$, i.e. $\forall s$, $\Delta(s) = \eta \Delta(s-1)$, which implies $\Delta(s) = \Delta(0)\, \eta^t$.

1.4 Naives vs sophisticates.

- Sophisticates understand the structure of the game and use backward induction.

  - In the example above a sophisticate understands that time 1 Self is not going to do the taxes and time 2 Self is going to do them, unless Self 0 does. So Self 0 chooses to do his taxes.

  - But the first best would be to force Self 1 to do the taxes.

  - You don't see too much commitment schemes in pratice.

  - Maybe they will be developed by the market, or maybe all consumers are naives.

- Naive thinks that future selfs will act according to his wishes.
  - Naives don't want commitment devices.

- Are people naives or sophisticates?
  - We see some commitment devices, e.g. mortgage is forced savings.

- Partial naives (O'Donoghue and Rabin, Doing it now or later, AER 1999)
  - Self t's preferences are $\left(1, \beta\delta, \beta\delta^2, ...\right)$ but Self t thinks that future selves have $\left(1, \hat{\beta}\delta, \hat{\beta}\delta^2, ...\right)$.
  - If $\hat{\beta} = \beta$ then the agent is sophisticated. If $\hat{\beta} = 1$ then the agent is naive.

1.5 Paradoxes with sophisticated hyperbolics

- Sophisticated hyperbolics have consumption that is a non-monotonic function of their wealth if there are borrowing constraints (Harris and Laibson, "Dynamic Choices of Hyperbolic Consumers", *Econometrica* 2004)

- This pushes very far the assumption of sophistication.

- That disappears if the environment is noisy enough (that smoothes out the ups and downs)

1.6 Continuous time hyperbolics

- Harris and Laibson: "Instantaneous gratification".
  - Agents maximize
$$\max \int_0^\infty \Delta(t)\, u(c_t)\, dt$$
    where $\Delta(t)$ equals $\Delta(t-dt)(1-\rho dt)$ with probability $1-\lambda dt$ and equals $\beta\Delta(t-dt)$ with probability λdt.
  - They have only one shock in a lifetime.

- So:
$$V = E\left[\int_t^{t+T} e^{-\rho(s-t)} u(c_s)\, ds + \beta \int_{t+T}^\infty e^{-\rho(s-t)} u(c_s)\, ds\right]$$

where T is a Poisson(λ) arrival time.

- One can do continuous time Bellman Equations.

- Nice paper by Luttmer and Mariotti (JPE 2003).