14.127 Lecture 7

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1 Learning in games

- Drew Fudenberg and David Levine, The Theory of Learning in Games

1.1 Fictitious play

- Let γ_t^i denotes frequencies of i's opponents play

$$\gamma_t^i\left(s_{-i}\right) = \frac{\text{number of times } s_{-i} \text{ was played till now}}{t}$$

- Player i plays the best response $BR\left(\gamma_t^i\right)$

- Big concerns:

  - Asymptotic behavior: do we converge or do we cycle?

  - If we converge, then to what subset of Nash equilibria?

- Caveat. Empirical distribution need not converge

1.2 Replicator dynamics

- Call $\theta_t^i\left(s^i\right) =$ fraction of players of type i who play s_i.

- Postulate dynamics

  – In discrete time

$$\vec{\theta}_{t+1}^i = \left(\theta_{t+1}^i\left(s_1\right), ..., \theta_{t+1}^i\left(s_n\right)\right) = \vec{\theta}_t^i + \lambda\left(BR\left(\vec{\theta}_t^{-i}\right) - \vec{\theta}_t^i\right)$$

  – In continuous time

$$\frac{d}{dt}\vec{\theta}_{t+1}^i = \lambda\left(BR\left(\vec{\theta}_t^{-i}\right) - \vec{\theta}_t^i\right)$$

- Then analyze the dynamics: chaos, cycles, fixed points

1.3 Experience weighted attraction model, EWA

- Camerer-Ho, Econometrica 1999

- Denote N_t =number of "observation equivalent" past responses such that

$$N_{t+1} = \rho N_t + 1$$

- Denote
  - $s_{ij}-$ strategy j of player i
  - $s_i(t)$ – strategy that i played at t
  - $\pi_i\left(s_{ij}, s_{-i}(t)\right)$ – payoff of i

- Perceived payoff with parameter $\phi \in [0, 1]$

$$A_{ij}(t) = \frac{1}{N_t}\left[\phi N_{t-1} A_{ij}(t-1) + \left(\delta + (1-\delta)\, 1_{s_{ij}=s_i(t)}\right) \pi_i \left(s_{ij}, s_{-i}(t)\right)\right]$$

- Attraction to strategy j

$$\rho_{ij}(t) = \frac{e^{\lambda A_{ij}(t)}}{\sum_{j'} e^{\lambda A_{ij'}(t)}}$$

- At time $t+1$ player i plays j with probability $\rho_{ij}(t)$

- Free parameters: $\delta, \phi, \rho, A_{ij}(0), N(0)$

- Some cases
  - If $\delta = 0$ – reinforcement learning (called also law of effect). You only reinforce strategies that you actually played
  - If $\delta > 0$ – law of simulated effect
  - If $\phi = 0$ – agent very forgetful

- **Proposition.** If $\phi = \rho$ and $\delta = 1$ then EWA is a belief-based model. Makes predictions of fictitious play.

- If $N(0) = \infty$ and $A_{ij}(0) =$ equilibrium payoffs then EWA agent is a dogmatic game theorist.

1.3.1 Functional EWA (f-EWA)

- Has just one parameters. Other endogenized. But still looks like data fitting.

- Camerer, Ho, and Chong working paper

- They look after parameters that fit all the games

- They R^2 is good

- Other people in this field: Costa-Gomez, Crawford, Erev

1.3.2 Critique

- Those things are more endogenous than postulated.

- E.g. fictitious play guy does not detect trends, but people do detect trends

- How do you model patterns, how do you detect patterns. Whole field of pattern recognition in cognitive psychology

- If you are interested in strategy number 1069, then strategy 1068 should benefit also. There is some smoothing

1.4 Cognitive hierarchy model of one-shot games

- Camerer - Ho, QJE forthcoming

- s_i^i – strategy j of player i and $\pi_i (s_i, s_{-i})$ – profit of player i

- Each level 0 player:
  - just postulates that other players play at random with probability $\frac{1}{N}$
  - best responses to that belief

- Each level k player:
  - thinks that there is a fraction of players of levels $h \in \{0, ..., k-1\}$
  - proportions are $g_k(h) = \frac{f(h)}{\sum_{h'=0}^{k-1} f(h')}$ and $g_k(h) = 0$ for $h \geq k$
  - k-players best response to this belief
- Camerer-Ho postulate a Poisson distribution for f with parameter τ,

$$f(k) = e^{-\tau} \frac{\tau^k}{k!}$$

with $Ek = \sum_{k \geq 0} k f(k) = \tau$.

- The authors calibrate to empirical data and find the average $\tau \simeq 1.5$.

1.5 An open problem – asymmetric information

- James has a plant with value V uniformly distributed over $[0, 100]$.
- James know V, you don't
- You are a better manager than James; the value to you is $\frac{3}{2}V$
- You can make a take it or leave it offer to James of x.
- What you would do?

- Empirically people offer between 50 and 75. But that is not the rational value.

- **Proposition.** The rational offer is 0.

- Proof. You offer x.

  - If $V > x$ then James refuses, and your payoff $W = 0$.

  - If $V \leq x$ then V is uniformly distributed between 0 and x. Hence your expected value is $W = \frac{3}{2} \cdot \frac{x}{2} - x = -\frac{x}{4}$.

  - Hence best you can do is set $x = 0$. QED

1.5.1 How to model people's choice?

- This game is not covered by cognitive hierarchy model. It is a single person decision problem.

- Maybe people approximate V by, for example, a unit mass at the mean $V = 50$?

- Other question. You own newspaper stand. You can buy newspaper for \$1 and have a chance to sell for \$4. There are no returns. The demand is uniform between 50 and 150. How many would you buy?

- Something along those lines will be in Problem Set 3.