14.127 Behavioral Economics. Lecture 11

Introduction to Behavioral Finance

Xavier Gabaix

April 22, 2004

2 Finance

Andrei Schleifer, Efficient Markets (book)

2.1 Closed end funds

- Fixed number of shares traded in the market

- The only way to walk away is to sell fund's share

- NAV – Net Asset Value – is the dollar value of single share computed as the value of the assets inside the shell net of liabilities divided by the number of the shares

- Discount $=$ (NAV-Share Price)/NAV

- The discount substantially decreased in early 80s.

2.2 Simplest limited arbitrage model

- One risky asset Q and one riskless asset (interest rate r)
- Two periods,
  - $t = 0$ trading,
  - $t = 1$ dividend $D = \bar{D} + \sigma z$ where $z \sim N(0, 1)$
- CARA expected utility $U(x) = -e^{-\gamma x}$.
- Buy q of the stock at price p and put $W - q$ in bonds.

- Payoff $x = qD + (1+r)(W - pq)$ and

$$EU = -Ee^{-\gamma(qD+(1+r)(W-pq))}$$

- Use $Ee^{a+bz} = e^{a+\frac{b^2}{2}}$ and get

$$EU = -e^{-\gamma\left(q\bar{D}+(1+r)(W-pq)\right)-\gamma\frac{q^2\sigma^2}{2}}$$

- Thus the agent maximizes $\max_q \gamma\left(q\bar{D} + (1+r)(W-pq)\right) + \frac{\gamma^2 q^2 \sigma^2}{2}$ and

$$q = \frac{\bar{D} - p(1+r)}{\gamma\sigma^2}$$

- This gives a downward sloping demand for stocks

- Imagine there are two types of agents
  - irrational buy q^I of stock
  - rational do maximization and buy q^R
- In equilibirum $q^I + q^R = Q$ and $p = \frac{\bar{D} - \gamma\sigma^2(Q - q^I)}{1+r}$
- Thus the price moves up with the number of irrational guys.
- This falsifies the claims that arbitragers will arbitrage influence of irrational guys away
- Risky stock reacts a lot to animal spirits

2.3 Noise trader risk in financial markets

- DeLong, Schleifer, Summers, Waldmann, JPE 1990

- Two types: noise traders (naives) and arbitragers (rational) with utility $U = e^{-2\gamma W}$

- Overlapping generations model

- NT have animal spirit shocks $\rho_t = E\rho_{t+1}$, $\rho_t \sim N\left(\rho^*, \sigma_\rho^2\right)$

- The stock gives dividend r at every period

- Call $\lambda_t^i =$ quantity of stock held by type $i \in \{NT, A\}$

- Demand

$$\max_{\lambda^i} E^i e^{-2\gamma\left(\lambda^i(p_{t+1}-(1+r)p_t)+(1+r)W\right)}$$

- For arbitragers

$$\lambda^A = \frac{r + E_t p_{t+1} - (1+r)\,p_t}{2\gamma\sigma^2_{t+1}}$$

- We postulate $E^{NT} p_{t+1} = E p_{t+1} + \rho_t$ and

$$\lambda^{NT} = \lambda^A + \frac{\rho_t}{2\gamma\sigma^2_{t+1}}$$

- Call μ – the fraction of noise traders, supply of stock is 1

- In general equilibrium

$$(1-\mu)\,\lambda_t^A + \mu\lambda_t^{NT} = 1$$

- Thus

$$\lambda_t^A + \frac{\mu\rho_t}{2\gamma\sigma_{t+1}^2} = 1$$

- Solving for price p_t

$$p_t = \frac{1}{1+r}\left(r + E_t p_{t+1} - 2\gamma\sigma_{t+1}^2 + \mu\rho_t\right)$$

- Solving recursively

$$E_{t-1}p_t = \frac{r}{1+r} + \frac{E_{t-1}p_{t+1}}{1+r}\frac{-2\gamma\sigma_{t+1}^2 + \mu\rho^*}{1+r}.$$

- In stationary equilibrium

$$E_{t-1}p_t = \frac{1}{r}\left(r - 2\gamma\sigma_{t+1}^2 + \mu\rho^*\right)$$

- Also,

$$\sigma_{t+1}^2 = \mathrm{Var}\left(\frac{1}{1+r}\mu\rho_t\right) = \frac{\mu^2}{(1+r)^2}\sigma_\rho^2$$

- Plugging in

$$p_t = 1 + \frac{\mu\left(\rho_t - \rho^*\right)}{1+r} + \frac{\mu\rho^*}{r} - 2\gamma\frac{\mu^2}{(1+r)^2}\sigma_\rho^2$$

with the second term reflecting bullish/bearish behavior, the third term reflecting average bullishness of NT, and the forth term reflecting riskiness of stock due to changes in animal spirits.

- Even a stock with riskless fundamentals is risky because of the presence of NT, and thus the price cannot be arbitraged away.

2.3.1 Problems:

- Price can be negative

- Deeper. Remind if there is no free lunch then we can write

$$p_t = E\left[\frac{M_{t+1}}{1+r}\left(p_{t+1} + D_{t+1}\right)\right]$$

  for a stochastic discount factor M_{t+1}.

  – Iterating

$$p_t = E\left[\frac{M_{t+1}}{1+r}\left(E\left[\frac{M_{t+2}}{1+r}\left(p_{t+2} + D_{t+1}\right)\right] + D_{t+1}\right)\right]$$

– In general

$$p_t = \sum_{i=1}^{k} \frac{r}{(1+r)^i} + \frac{1}{(1+r)^k} E\left[M_{t+1} ... M_{t+k} p_{t+k}\right]$$

$$= 1 - \frac{1}{(1+r)^k} + \frac{1}{(1+r)^k} E\left[M_{t+1} ... M_{t+k} p_{t+k}\right]$$

– If you constrain the price to be positive, then

$$p_t \geq 1$$

– One reference on this, Greg Willard et al (see his Maryland website)