

# 14.128 Dynamic Optimization and Economic Applications (Recursive Methods)

Economics Department  
Spring 2003

The unifying theme of this course is best captured by the title of our main reference book: 'Recursive Methods in Economic Dynamics'. We start by covering deterministic and stochastic dynamic optimization using dynamic programming analysis. We then study the properties of the resulting dynamic systems. Finally, we will go over a recursive method for repeated games that has proven useful in contract theory and macroeconomics. We shall stress applications and examples of all these techniques throughout the course.

The main reference for the course is (hereafter, SLP):

*Stokey, Nancy L. and Robert E. Jr. Lucas with Edward C. Prescott, 1989, Recursive Methods in Economic Dynamics, Cambridge, MA: Harvard University Press.*

We will follow SLP's exposition as closely as possible, departing only to add more recent developments and applications when needed.

Although some homework assignments will seek to introduce you to solving problems numerically, numerical methods are not a main part of this course. I highly recommend Kenneth Judd's 'Numerical Methods in Economics' book as a complement of the material of this course.

The course grade will be based on problem set homework (30%) and a final exam (70%).

TA sessions are once a week and will be used mainly to go over problems and some additional material not covered in the lectures.

## **Course Outline (each topic approximately 1-1½ week)**

Topic 1: Preliminaries; Euler Equations and Transversality Conditions; Principle of Optimality [SLP Chapters 3-4.1]

Topic 2: Bounded Returns; Differentiability of Value Function; Homogenous and Unbounded Returns; Applications [SLP Chapters 4-5]

Topic 3: Deterministic Global and Local Dynamics [SLP Chapter 6]

Topic 4: Stochastic Dynamic Programming; Applications; Markov Chains [SLP Chapter 9-11]

Topic 5: Weak Convergence; Applications [SLP Chapters 12-13]

Topic 6: Repeated Games and Dynamic Contracts (APS)

Topic 7 (if time permits): Continuous-Time Dynamic Programming and Hamilton-Jacobi-Bellman PDE Equations [Fleming and Soner, Chapter 1].

## References

### **Euler Equations and Transversality: Sufficiency and Necessity**

- <sup>J</sup>Weitzman, M. L. (1973) “Duality Theory for Infinite Horizon Convex Models”, *Management Science* 19, 783-789.
- \*Benveniste and Scheinkman “Duality Theory for Dynamic Optimization Models of Economics: The Continuous Time Case” *Journal of Economic Theory*, v. 27, 1-19.
- \*Kamihigashi, Takashi “Necessity of Transversality Conditions for Infinite Horizon Problems” *Econometrica* v69, n4 (July 2001): 995-1012
- <sup>MIT</sup>Kamihigashi, Takashi “A Simple Proof of the Necessity of the Transversality Condition” *Economic Theory* v20, n2 (September 2002): 427-33
- \*Araujo, A. and J. A. Scheinkman, “Maximum Principle and Transversality Condition for Concave Infinite Horizon Economic Models” *Journal of Economic Theory* v30, n1 (June 1983): 1-16
- <sup>J</sup>Michel, Philippe “On the Transversality Condition in Infinite Horizon Optimal Problems” *Econometrica* v50, n4 (July 1982): 975-85
- <sup>J</sup>Michel, Philippe “Some Clarifications on the Transversality Condition” *Econometrica* v58, n3 (May 1990): 705-23
- \*Leung, Siu Fai “Transversality Condition and Optimality in a Class of Infinite Horizon Continuous Time Economic Models”, *Journal of Economic Theory*, v54, n1 (June 1991): 224-33

### **Differentiability of the Value Function**

- \*Milgrom, Paul and Ilya Segal (2002) “Envelope Theorems for Arbitrary Choice Sets”, *Econometrica*, v. 70, n. 2 (March), 583-601.

### **Differentiability of the Policy Function**

- <sup>J</sup>Araujo, A. “The Once but Not Twice Differentiability of the Policy Function” *Econometrica* v59, n5 (September 1991): 1383-93
- <sup>J</sup>Araujo, A. and J. A. Scheinkman (1977) “Smoothness, Comparative Dynamics, and the Turnpike Property”, *Econometrica*, v. 45, n. 3 (April).
- <sup>J</sup>Santos, Manuel S. (1991) “Smoothness of the Policy Function in Discrete Time Economic Models”, *Econometrica*, v. 59, n. 5.

### **Homogenous Dynamic Programming**

- <sup>MIT</sup>Alvarez, Fernando and Nancy L. Stokey (1998) “Dynamic Programming with Homogenous Functions”, *Journal of Economic Theory*, 82, 167-189.

### **Complicated Dynamics from Dynamic Optimization Problems**

- \*Boldrin, Michele and Luigi Montrucchio (1986) “On the Indeterminacy of Capital Accumulation Paths”, *Journal of Economic Theory*, 40, 26-29.
- <sup>MIT</sup>Mitra, Tapan (1998) “On the Relationship between Discounting and Complicated Behavior in Dynamic Optimization Models”, *Journal of Economic Behavior and Organizations*, v. 33, 421-434.

### **Repeated Games and Dynamic Contracts**

<sup>J</sup>Abreu, Dilip, David Pearce and Ennio Stacchetti (1990) “Towards a Theory of Infinitely Repeated Games with Discounting”, *Econometrica*, v. 58, n. 5, 1041-1063.

Ljungqvist, Lars and Thomas Sargent, ‘Recursive Macroeconomic Theory’, Chapter 16.

\*Phelan, Christopher and Ennio Stacchetti (1999) “Sequential Equilibria in a Ramsey Tax Model”, *Econometrica* v69, n6 (November 2001): 1491-1518

\*Athey, Susan, Andrew Atkeson, Patrick J. Kehoe (2001) “On the optimality of transparent monetary policy”, Federal Reserve Bank of Minneapolis, Working Paper Number 613

\*Athey, Susan, Kyle Bagwell and Chris Sanchirico (2002) “Collusion and Price Rigidity”, manuscript

\*Jonathan Levin (2002) “Relational Incentive Contracts” manuscript, Stanford University

### **Continuous-Time Dynamic Programming**

Fleming, Wendell Helms and H. Mete Soner (1993) “Controlled Markov Processes and Viscosity Solutions”, *Applications of Mathematics*, vol. 25, Springer-Verlag [chapter 1]