1 DG monopoly with Fixed Types

Buyer-Seller: R-N, $\delta \leq 1$; 2 periods.

Buyer: v_i per period, $0 < v_L < v_H$,

x_{it} is prob buyer i consumes in period t.

Seller: $c = 0$, $\Pr(v_H) = \beta$.

- Full-Commitment:

Menu $(X_i, T_i)_{i=L,H}$, where $X_i = x_{i1} + \delta x_{i2}$.

Seller: $(1-\beta)T_L + \beta T_H \to_{X_i,T_i} \max$, s.t.

$$\begin{cases} v_i X_i - T_i \geq 0, & i = L, H \\ v_i X_i - T_i \geq v_i X_j - T_j, & i, j = L, H \end{cases}$$
$$0 \leq x_{it} \leq 1, \qquad i = L, H; t = 0, 1.$$

IRL, ICH are binding:

$(1-\beta)v_L X_L + \beta(v_H X_H - (v_H - v_L)X_L) \to_{X_L,X_H} \max.$

Thus, $X_H = 1 + \delta \equiv \Delta$. Set $\beta^* \equiv \frac{v_L}{v_H}$.

If $\beta < \beta^*$, $X_L = \Delta$, $T_L = T_H = v_L\Delta$ $(P = v_L)$.

Otherwise, $X_L = 0 = T_L$, $T_H = v_H\Delta$.

- Selling DG: No-Commitment. $(\beta > \beta^*)$

P_t is price in period t.

If object is sold in period t, it is consumed in each period thereafter.

Let $\beta_t = \Pr(i = H|t)$, $\beta_1 = \beta$, $\beta_2 = \beta_2(I_1)$, where I_1 is the outcome (information set) of period 1.

Period 2 (as before) depends on $\beta_2 \gtrless \beta^*$.

Period 1: L gets zero surplus, accepts $P_1 \leq v_L \Delta$.

Type H decision depends on Exp of $t = 2$:

$P_2 = v_H \rightarrow H$ accepts $P_1 \leq v_H \Delta$.

$P_2 = v_L \rightarrow H$ accepts $P_1 \leq v_H + \delta v_L \equiv P^*$.

Seller's options: (1) $P_1 = ER = v_L \Delta$.

(2) $P_2 = v_L$, $P_1 = P^*$,

$ER = (1-\beta)\delta v_L + \beta P^* = \beta v_H + \delta v_L$ $(> ER^{(1)})$.

(3) (mixed str) Seller rnds over P_2, $\sigma = \Pr(P_2 = v_H)$; buyer H rnds over buying in $t = 1$ (γ is prob).

Seller indiff: $v_L = \beta_2 v_H$, thus

$$\beta_2 = \beta^* = \frac{\beta(1-\gamma)}{\beta(1-\gamma)+(1-\beta)}; \quad \gamma = \frac{\beta-\beta^*}{\beta(1-\beta^*)}.$$

Buyer indiff:

$$v_H \Delta - P_1 = \delta\,(1-\sigma)\,(v_H - v_L); \quad \sigma = 1 - \frac{v_H \Delta - P_1}{\delta(v_H - v_L)}.$$

Seller's revenue:

$$\beta\gamma P_1 + \delta\,[\beta(1-\gamma)(\sigma v_H + (1-\sigma)v_L) + (1-\beta)(1-\sigma)v_L]$$

Substitute either P_1 or σ. Linear objective.

Solution: $P_1 = v_H \Delta$, $\sigma = 1$.

$ER = \beta v_H(\gamma\Delta + (1-\gamma)\delta)$.

When $\beta \to \beta^*$, $\gamma \to 0$, $ER \to \delta\beta v_H$. No randomizing.

When $\beta \to 1$, $\gamma \to 1$, $ER \to \beta\Delta v_H$. Randomizing is preferred.

Note, by "randomizing" seller still sells only to a high-valued buyer, but, with no commitment, sometimes no sale happens in period 1.

- Renting without Commitment.

Buyer pays R_t to consume in period t.

This would help if types were not fixed: with iid types seller can optimize each period, while selling still suffers competition from future selves.

(+) Rachet effect: cannot commit not to raise the price in period 2.

Period 2: $R_2 = v_H (= v_L)$ if $\beta_2 > (<) \beta^*$.

Two β's possible (reject/accept!). Here, they are the same.

Period 1: (1) $R_1 = v_L$, $R_2 = v_H$, $ER = v_L + \delta\beta v_H$.

(2) Separating regime: $v_H - R_1 \geq \delta(v_H - v_L)$. $ER = \beta(v_H - \delta(v_H - v_L)) + \delta(\beta v_H + (1-\beta)v_L) = \beta v_H + \delta v_L > ER^{(1)}$ (here, and from now on, β_t is probability of v_H conditional on rejection.)

(3) Semi-separating regime: H rents with prob $\gamma = \frac{\beta-\beta^*}{\beta(1-\beta^*)}$, seller is indifferent between setting R_2 to v_L or v_R after rejection.

Seller's probability of $R_2 = v_H$ is σ.

As before: $\sigma = 1$, $R_1 = v_H$. ER the same.

- More than two periods. β_t is prob of v_H conditional on rejected before.

Suppose there exists $t < T$, such that $\beta_t < \beta^*$, consider lowest possible t. Then, $R_\tau = v_L$ for all $\tau \geq t$.

Consider period $t-1$. Since $\beta_{t-1} \geq \beta^*$, there are high types that pay R_{t-1} and signal who they are.

To do so, $v_H - R_{t-1} \geq (v_H - v_L)\delta(1+\delta+\cdots+\delta^{T-t})$.

If, however, $\delta(1+\delta) > 1$, $R_{t-1} < v_L$ (cannot happen). Then, $\beta_t \geq \beta^*$ for all t. Not much revelation possible.

Suppose β is close to β^*.

Selling: Separation is optimal with $T = 2$. If $T = 3$, the seller can set $P_1 = v_H + (\delta + \delta^2)v_L$, $P_2 = (1+\delta)v_L$.

Renting when $T = 3$:

(1) Set $R_1 > v_L$, so that $\beta_2 = \beta^*$. Remaining payoff is $(1+\delta)v_L$. In period 1, $R_1 \leq v_H$, and probability of sale is $< \beta$. Worse than selling.

(2) $R_1 = v_L$, and then two-periods full separation. Worse than selling again because, $\beta > \beta^*$.

- Renegotiation-proof contracts.

Sequential Pareto-Optimality.

$T = 2$, PO means $P_2 = v_H (= v_L)$ if $\beta_2 > (<) \beta^*$. Exactly the same requirement as with no-commitment.

Previous cases can be represented as renegotiation-proof contracts.