1 Dynamic Moral Hazard

- Intertemporal risk-sharing
- Better information (output, actions, consumption)
- Larger games (action spaces)
- Generic complexity (?spot contracting)

Simple M (separable): $t = 1, 2$.

$a \in A$, $\#Q = n$, $\Pr(q_i^t = q_i | a = a_t) = p_i(a_t) > 0$.

Agent: $u(c) - \psi(a)$ (in each t), $\lim_{c \downarrow \bar{c}} u(c) = -\infty$.

Principal: $V(q - w)$.

Contracting: $t = 1$: $\{a_1, w_1(q_i^1), a_2(q_i^2), w_2(q_i^1, q_j^2)\}$. (RP)

- No savings or borrowing.

Principal chooses: w_i, w_{ij}; Agent: α, a_i.

$\max_{w_i, w_{ij}} \sum_i p_i(\alpha) \left[V(q_i^1 - w_i) + \sum_j p_j(a_i) V(q_i^2 - w_{ij}) \right]$,

s.t. $\alpha, a_i \in \arg\max AG(\alpha, a_i, w_i, w_{ij})$, and IR.

Euler equation:

$$\frac{V'(q_i^1 - w_i)}{u'(w_i)} = \sum_j p_j(a_i) \left[\frac{V'(q_i^2 - w_{ij})}{u'(w_{ij})} \right]$$

When $V' = const$, we have "smoothing"

$$\frac{1}{u'(w_i)} = \sum_j p_j(a_i) \left[\frac{1}{u'(w_{ij})} \right].$$

Two observations: (1) Optimal contract has memory,

No memory would imply RHS is constant for all i, perfect insurance in period 1, wrong incentives.

(2) Agent wants to save (and so the contract is "front-loaded").

$\frac{\partial EU}{\partial s} = \sum_j p_j(a_i)u'(w_{ij}) - u'(w_i) \geq 0$ (Jensen's inequality).

- Monitored savings

Add t_i, s_i (principal, agent)'s savings.

The above contract can be achieved without history-dependent wages, and, so, is spot-implementable.

Set: $c_{ij} = w_{ij} = w_j + s_i$, $w_i = c_i - s_i$.

Problem separates to: incentive provision and consumption smoothing.

- Free savings.

Example: Effort in $t = 2$, consumption in both periods (borrowing in the first period)

$a \in \{H, L\}$, $\psi(H) = 1, \psi(L) = 0$.

$q \in \{0, 1\}$, $p_H = p_1(H) > p_L > 0$.

Suppose $a^* = H$. Contract (w_0, w_1).

Let c^j be consumption with planned $j = H, L$.

$c^j \in \arg\max_c u(c) + p_j u(w_1 - c) + (1 - p_j)u(w_0 - c)$.

We have

$$
\begin{aligned}
& u(c^H) + p_H u(w_1 - c^H) + (1 - p_H)u(w_0 - c^H) - 1 = \\
& \quad = u(c^L) + p_L u(w_1 - c^L) + (1 - p_L)u(w_0 - c^L) \\
& \quad > u(c^H) + p_L u(w_1 - c^H) + (1 - p_L)u(w_0 - c^H)
\end{aligned}
$$

Thus $ICH2$ is slack. Room for renegotiation (unless CARA)

1.1 T-period Problem

Subcases:

- Repeated Output (better statistical inference)
- Repeated Actions (multitask in time)
- Repeated Consumption (consumption smoothing)
- Repeated Actions and Output (consumption at the end)
- Infinitely repeated Actions, Output, and Consumption.