1 Moral Hazard: Multiple Agents

- Multiple agents (firm?)
  - Partnership: Q jointly affected
  - Individual q_i's. (tournaments)
- Common shocks, cooperations, collusion, monitoring.

Agents: $i = 1, \ldots, n$.

$U_i(w_i, a_i) = u_i(w_i) - \psi_i(a_i)$

Efforts: $a = (a_1, \ldots, a_n)$, with $a_i \in [0, \infty)$

Output $q = (q_1, \ldots, q_n) \sim F(q|a)$.

Principal: R-N.

1.1 Moral Hazard in a Team

Holmstrom(82) Deterministic Q.

Output $Q(a) \sim F(q|a)$, $\frac{\partial Q}{\partial a_i} > 0$, $\frac{\partial^2 Q}{\partial a_i^2} < 0$,

$dq_{ij} = \frac{\partial^2 Q}{\partial a_i \partial a_j} \geq 0$, $(dq)_{ij}$–negative definite.

Agents: $u_i(w) = w$.

Partnership $w(Q) = \{w_1(Q), \ldots, w_n(Q)\}$,

such that for all Q, $\sum_{i=1}^n w_i(Q) = Q$.

Problem: free-riding (someone else works hard, I gain)

First-best: $\frac{\partial Q(a^*)}{\partial a_i} = \psi'(a_i^*)$.

Agents' choices: FOC:

$$\frac{dw_i[Q(a_i, a^*_{-i})]}{dQ} \frac{\partial Q(a_i, a^*_{-i})}{\partial a_i} = \psi'(a_i)$$

? Nash (a_i^*) = FB (a_i^*)?

Locally:

$\frac{dw_i[Q(a_i,a^*_{-i})]}{dQ} = 1$, thus $w_i(Q) = Q + C_i$.

Budget $\sum_{i=1}^n w_i(Q) = Q$ for all (!) Q.

This requires a third party: *budget breaker*

Let $z_i = -C_i$—payment from agent i.

Thus $\sum_{i=1}^n z_i + Q(a^*) \geq nQ(a^*)$

and $z_i \leq Q(a^*) - \psi_i(a_i^*)$.

At F-B: $Q(a^*) - \sum_{i=1}^n \psi_i(a_i^*) > 0$.

Thus $\exists z = (z_1, \ldots, z_n)$.

Note, b-b looses from higher Qs.

Comments: b-b is a residual claimant (in fact each agent is a residual claimant in a certain interpretation (!)).

Not the same as Alchian & Demsetz (equity for manager's incentives to monitor agents properly).

? Other ways to support first-best?

Mirrlees contract: reward (bonus) b_i if $Q = Q(a^*)$,

penalty k otherwise. (bonuses for certain targets)

As long as $b_i - \psi_i(a_i^*) \geq -k$, F-B can be supported, moreover if b's and k exist so that $Q(a^*) \geq \sum_{i=1}^n b_i$, no b-b needed.

Interpretation: Debt financing by the firm.

Firm commits to repay debts of $D = Q(a^*) - \sum b_i$, and b_i to each i.

If cannot, creditors collect Q and each employer pays k. (Hm...)

Issues: (1) Multiple equilibria (like in all coordination-type games, and in Mechanism-Design literature). No easy solution unless

(2) actions of others are observed by agents, and the principal can base his compensation on everyone's reports. Not a problem with Holmstrom though (Positive effort of one agent increases effort from others).

(3) Deterministic Q.

1.2 Special Examples of F-B (approx) via different schemes

Legros & Matthews ('93), Legros & Matsushima ('91).

- Deterministic Q, finite A's, detectable deviations.

Say, $a_i \in \{0, 1\}$. And $Q^{fb} = Q(1, 1, 1)$.

Let $Q_i = Q(a_i = 0, a_{-i} = (1, 1))$.

Suppose $Q_1 \neq Q_2 \neq Q_3$.

Shirker identified and punished (at the benefit of the others).

Similarly, even if $Q_1 = Q_2 \neq Q_3$.

- Approx. efficiency, $n = 2$.

Idea: use one agent to monitor the other (check with prob ε).

$a_i \in [0, \infty)$ $Q = a_1 + a_2$, $\psi_i(a_i) = a_i^2/2$.

F-B: $a_i^* = 1$.

L & M propose: agent 1 chooses $a_1 = 1$ with $pr = 1-\varepsilon$.

When $Q \geq 1$,

$$\begin{cases} w_1(Q) = (Q-1)^2/2 \\ w_2 = q - w_1(Q). \end{cases}$$

when $Q < 1$,

$$\begin{cases} w_1(Q) = Q + k \\ w_2(Q) = 0 - k. \end{cases}$$

Check: Agent 1. Set $a_2 = 1$,

$\max_a \left[\frac{((a+1)-1)^2}{2} - \frac{a^2}{2}\right] = 0.$

Agent 2. $a_2 \geq 1 \mapsto Q \geq 1$. Implies $a_2^* = 1$, $U_2 = 1 - \varepsilon/2$.

$a_2 < 1$ guarantees $Q < 1$ with prob. ε.

Obtain $a_2^* = \frac{1}{2}$, and $U_2 = \frac{5}{4} - \varepsilon k$.

For, $k \geq \frac{1}{2} + \frac{1}{4\varepsilon}$, $a_2^* = 1$ is optimal.

- Random output. Cremer & McLean works. (conditions?)

1.3 Observable individual outputs

$$\begin{aligned} q_1 &= a_1 + \varepsilon_1 + \alpha\varepsilon_2, \\ q_2 &= a_2 + \varepsilon_2 + \alpha\varepsilon_1. \end{aligned}$$

$\varepsilon_1, \varepsilon_2 \sim$ iid $N(0, \sigma^2)$.

CARA agents: $u(w,a) = -e^{-\mu(w-\psi(a))}$, $\psi(a) = \frac{1}{2}ca^2$.

Linear incentive schemes:

$$\begin{aligned} w_1 &= z_1 + v_1 q_1 + u_1 q_2, \\ w_2 &= z_2 + v_2 q_2 + u_2 q_1. \end{aligned}$$

No relative performance weights: $u_i = 0$,

Principal: $\max_{a,z,v,u} E(q-w)$,

subject to $E\left[-e^{-\mu(w-\psi(a))}\right] \geq u(\bar{w})$.

Define $\hat{w}(a)$, as $-e^{-\mu\hat{w}(a)} = E\left[-e^{-\mu(w-\psi(a))}\right]$.

Agent's choice: $a \in \arg\max \hat{w}(a)$.

$E(e^{a\varepsilon}) = e^{a^2\sigma^2/2}$, for $\varepsilon \sim N(0, \sigma^2)$.

(back to General case) Agent i

$$V(w_1) = Var(v_1(\varepsilon_1 + \alpha\varepsilon_2) + u_1(\varepsilon_2 + \alpha\varepsilon_1))$$

$$= \sigma^2\left[(v_1 + \alpha u_1)^2 + (u_1 + \alpha v_1)^2\right]$$

Then, agent's problem:

$$\max_a \left\{ \begin{array}{c} z_1 + v_1 a + u_1 a_2 - \frac{1}{2}ca^2 - \\ -\frac{\mu\sigma^2}{2}\left[(v_1 + \alpha u_1)^2 + (u_1 + \alpha v_1)^2\right] \end{array} \right\}.$$

Solution $a_1^* = \frac{v_1}{c}$ (as in one A case).

$$\hat{w}_1 = z_1 + \frac{1}{2}\frac{v_1^2}{c} + \frac{u_1 v_2}{c} - \frac{\mu\sigma^2}{2}\left[(v_1 + \alpha u_1)^2 + (u_1 + \alpha v_1)^2\right].$$

Principal: $\max_{z_1,v_1,u_1}\left\{\frac{v_1}{c} - \left(z_1 + \frac{v_1^2}{c} + \frac{u_1 v_2}{c}\right)\right\}$

s.t. $\hat{w}_1 \geq \bar{w}$

Principal:

$$\max_{v_1,u_1} \left\{ \frac{v_1}{c} - \frac{1}{2}\frac{v_1^2}{c}\frac{\mu\sigma^2}{2} - \left[(v_1 + \alpha u_1)^2 + (u_1 + \alpha v_1)^2 \right] \right\}.$$

To solve: (1) find u_1 to minimize sum of squares (risk)

(2) Find v_1 (trade-off) risk-sharing, incentives

Obtain $u_1 = -\frac{2\alpha}{1+\alpha^2} v_1$.

The optimal incentive scheme reduce agents' exposure to common shock.

$v_1 = \frac{1+\alpha^2}{1+\alpha^2+\mu c\sigma^2(1-\alpha^2)^2}$.

1.4 Tournaments

Lazear & Rosen ('81)

Agents: R-N, no common shock.

$q_i = a_i + \varepsilon_i$. $\varepsilon \sim F(\cdot)$, $E = 0$, $Var = \sigma^2$.

Cost $\psi(a_i)$.

F-B: $1 = \psi'(a^*)$.

$w_i = z + q_i$.

$z + E(q_i) - \psi(a^*) = z + a^* - \psi(a^*) = \bar{u}$.

Tournament: $q_i > q_j \rightarrow$ prize W, both agents paid z.

Agent: $z + pW - \psi(a_i) \rightarrow_{a_i}$ max.

$p = Pr(q_i > q_j) =$

$= Pr(a_i - a_j > \varepsilon_j - \varepsilon_i) = H(a_i - a_j).$

$E_H = 0$, $Var_H = 2\sigma^2$.

FOC: $W \frac{\partial p}{\partial a_i} = \psi'(a_i)$.

$Wh(a_i - a_j) = \psi'(a_i)$.

Symmetric Nash: (+FB): $W = \frac{1}{h(0)}$.

$z + \frac{H(0)}{h(0)} - \psi(a^*) = \bar{u}$.

Result: Same as FB with wages.

Extension: multiple rounds, prizes progressively increasing.

Agents: Risk-averse+Common Shock.

Trade-off between (z, q) contracts and tournaments.

1.5 Cooperation and Competition

- Inducing help vs Specialization
- Collusion among agents
- Principal-auditor-agent

Itoh ('91)

2 agents: $q_i \in \{0, 1\}$, $(a_i, b_i) \in [0, \infty) \times [0, \infty)$.

$U_i = u_i(w) - \psi_i(a_i, b_i)$, $u_i(w) = \sqrt{w}$.

$\psi_i(a_i, b_i) = a_i^2 + b_i^2 + 2ka_ib_i$, $k \in [0, 1]$.

$\Pr(q_i = 1) = a_i(1 + b_i)$.

Contract: $w^i = (w^i_{jk})$, w^i_{jk} – payment to i when $q_i = j$, $q_{-i} = k$.

No Help: $b_i = 0$.

$w_0 = 0$, $a_i(1 - w_1) \to_{w_1} \max$,

s.t, $a_i = \frac{1}{2}\sqrt{w_1}$ (IC) and IR is met. ...

Getting Help: Agent i solves (given $a_j, b_j, w, w_{11} > w_{10}, w_{01} > w_{00} = 0$.)

$$a(1+b_j)a_j(1+b)\sqrt{w_{11}} + (1 - a(1+b_j))a_j(1+b)\sqrt{w_{01}} + a(1+b_j)(1-a_j(1+b))\sqrt{w_{10}} - a^2 - b^2 - 2kab \to \max_{a,b}$$

FOC+symm: consider $\left(\frac{\partial}{\partial b}\right)$

$$a^2(1+b)\left(\sqrt{w_{11}} - \sqrt{w_{10}}\right) + a(1-a(1+b))\sqrt{w_{01}} = 2(b+ak)$$

If (as in No Help) $w_{11} = w_{10}$, $w_{01} = 0$, and $k > 0$, we have $RHS = 0$, $LHS > 0$ for any $b \geq 0$.

Therefore, need to change w significantly to get any b close to 0. (Even to get $b = 0$ with $FOC_b = 0$)

By itself (ignoring change in a) and if b^* is small, and since change in w increases risk, it is costly for the principal to provide these incentives.

Even if a adjusts, since it is different from the first-best for the principal with $b = 0$, the principal looses for sure.

Thus, if k is positive, there is a discontinuity at $b = 0$, thus "a little" of help will not help: for all $b < b^*$ principal is worse-off.

For $k = 0$, help is always better.

Two-step argument: 1. If $a^{help} \geq a^{b=0}$, marginal cost of help is of second order, always good.

2. Show that $a^{help} \geq a^{b=0}$.

1.6 Cooperation and collusion.

CARA agents: $u(w,a) = -e^{-\mu_i(w_i - \psi_i(a))}$.

$q_i = a_i + \varepsilon_i$, $(\varepsilon_1, \varepsilon_2) \sim N(0, V)$, where $V = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}$, $\rho = \sigma_{12}/(\sigma_1\sigma_2)$.

Linear incentive schemes:

$$\begin{aligned} w_1 &= z_1 + v_1 q_1 + u_1 q_2, \\ w_2 &= z_2 + v_2 q_2 + u_2 q_1. \end{aligned}$$

- No side contracts ($CE_2(a_1, a_2)$ analogously):

$$\begin{aligned} CE_1(a_1 a_2) &= z_1 + v_1 a_1 + u_1 a_2 - \psi_1(a_1) \\ &\quad -\frac{\mu_1}{2}(v_1^2\sigma_1^2 + u_1^2\sigma_2^2 + 2v_1 u_1 \sigma_{12}) \end{aligned}$$

Principal (RN):

$(1 - v_1 - u_2)a_1 + (1 - u_1 - v_2)a_2 - z_1 - z_2 \rightarrow \max$

s.t (a_1^*, a_2^*)–NE in efforts, and $CE_i \geq 0$.

Individual choices: $v_i = \psi_i'(a_i)$.

u are set to minimize risk-exposure: $u_i = -v_i \frac{\sigma_i}{\sigma_j}\rho$.

Total risk exposure: $\sum_{i=1}^2 \mu_i \left[v_i^2 \sigma_i^2 (1-\rho) \right]$.

- Full side-contracting:

- (?) Enough to consider contracts on (a_1, a_2).

- Problem reduces to a single-agent problem with $\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2}$, with costs $\psi(a_1, a_2) = \psi_1(a_1) + \psi_2(a_2)$.

- Full side contracting dominates no s-c iff $\rho \leq \rho^*$. (cooperation vs relative-performance evaluation).

- MD schemes.

1.7 Supervision and Collusion

Principal: $V > 1$,

Agent: cost $c \in \{0, 1\}$, Pr$(c = 0) = \frac{1}{2}$.

Monitor: cost z, Proof y^*, Pr$(y^*|c = 0) = p)$.

Assume: $V > 2$ (so $P = 1$ is optimal without monitor)

With monitor (no collusion)

$\frac{1}{2}pV + \left(1 - \frac{1}{2}p\right)(V - 1) - z$

Compare to $V - 1$.

Collision: Agent-Monitor: $T_{agent} \rightarrow (kT)_{monitor}$, $k \leq 1$.

$\max T = 1$.

Principal: reward Monitor for y^* with $w \geq k$.

(Punish when there is not y^*?)

Suppose not, that is $\frac{1}{2}pk > z$. (and thus suppose that $w_{mon} = 0$)

Principal: $\frac{1}{2}p(V - k) + \left(1 - \frac{1}{2}p\right)(V - 1)$.

- No gain for allowing collusion
- If k is random, then possible.